

Name _____

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.**Solve the system of equations.**

1) $x_1 - x_2 + 3x_3 = -8$ 1) _____

$2x_1 + x_3 = 0$

$x_1 + 5x_2 + x_3 = 40$

A) (8, 8, 0)

B) (-8, 0, 0)

C) (0, 8, 0)

D) (0, -8, -8)

Answer: C

2) $x_1 + 3x_2 + 2x_3 = 11$ 2) _____

$4x_2 + 9x_3 = -12$

$x_3 = -4$

A) (1, -4, 6)

B) (-4, 6, 1)

C) (1, 6, -4)

D) (-4, 1, 6)

Answer: C

3) $x_1 - x_2 + 8x_3 = -107$ 3) _____

$6x_1 + x_3 = 17$

$3x_2 - 5x_3 = 89$

A) (5, 8, -13)

B) (5, -8, -13)

C) (-5, 8, 13)

D) (-5, -8, 13)

Answer: A

4) $4x_1 - x_2 + 3x_3 = 12$ 4) _____

$2x_1 + 9x_3 = -5$

$x_1 + 4x_2 + 6x_3 = -32$

A) (2, -7, -1)

B) (2, 7, -1)

C) (2, 7, 1)

D) (2, -7, 1)

Answer: A

5) $x_1 + x_2 + x_3 = 6$ 5) _____

$x_1 - x_3 = -2$

$x_2 + 3x_3 = 11$

A) (1, 2, 3)

B) No solution

C) (-1, 2, -3)

D) (0, 1, 2)

Answer: A

6) $x_1 + x_2 + x_3 = 7$ 6) _____

$x_1 - x_2 + 2x_3 = 7$

$5x_1 + x_2 + x_3 = 11$

A) (1, 4, 2)

B) (1, 2, 4)

C) (4, 1, 2)

D) (4, 2, 1)

Answer: B

7) $x_1 - x_2 + x_3 = 8$ 7) _____
 $x_1 + x_2 + x_3 = 6$
 $x_1 + x_2 - x_3 = -12$
 A) (2, -1, 9) B) (-2, -1, 9) C) (-2, -1, -9) D) (2, -1, -9)
 Answer: B

8) $5x_1 + 2x_2 + x_3 = -11$ 8) _____
 $2x_1 - 3x_2 - x_3 = 17$
 $7x_1 + x_2 + 2x_3 = -4$
 A) (0, -6, 1) B) (-3, 0, 4) C) (0, 6, -1) D) (3, 0, -4)
 Answer: A

9) $7x_1 + 7x_2 + x_3 = 1$ 9) _____
 $x_1 + 8x_2 + 8x_3 = 8$
 $9x_1 + x_2 + 9x_3 = 9$
 A) (-1, 1, 1) B) (0, 0, 1) C) (1, -1, 1) D) (0, 1, 0)
 Answer: B

10) $2x_1 + x_2 = 0$ 10) _____
 $x_1 - 3x_2 + x_3 = 0$
 $3x_1 + x_2 - x_3 = 0$
 A) (1, 0, 0) B) (0, 1, 0) C) No solution D) (0, 0, 0)
 Answer: D

Determine whether the system is consistent.

11) $x_1 + x_2 + x_3 = 7$ 11) _____
 $x_1 - x_2 + 2x_3 = 7$
 $5x_1 + x_2 + x_3 = 11$
 A) Yes B) No
 Answer: A

12) $5x_1 + 2x_2 + x_3 = -11$ 12) _____
 $2x_1 - 3x_2 - x_3 = 17$
 $7x_1 + x_2 + 2x_3 = -4$
 A) No B) Yes
 Answer: B

13) $4x_1 - x_2 + 3x_3 = 12$ 13) _____
 $2x_1 + 9x_3 = -5$
 $x_1 + 4x_2 + 6x_3 = -32$
 A) Yes B) No
 Answer: A

- 14) $2x_1 + x_2 = 0$ 14) _____
 $x_1 - 3x_2 + x_3 = 0$
 $3x_1 + x_2 - x_3 = 0$
 A) Yes B) No
 Answer: A
- 15) $x_1 + x_2 + x_3 = 6$ 15) _____
 $x_1 - x_3 = -2$
 $x_2 + 3x_3 = 11$
 A) No B) Yes
 Answer: B
- 16) $x_1 - x_2 + 4x_3 = 15$ 16) _____
 $-4x_1 + 4x_2 - 16x_3 = 4$
 $x_1 + 4x_2 + x_3 = 0$
 A) No B) Yes
 Answer: A
- 17) $x_1 + x_2 + x_3 = 7$ 17) _____
 $x_1 - x_2 + 2x_3 = 7$
 $2x_1 + 3x_3 = 15$
 A) Yes B) No
 Answer: B
- 18) $x_1 + 3x_2 + 2x_3 = 11$ 18) _____
 $4x_2 + 9x_3 = -12$
 $x_1 + 7x_2 + 11x_3 = -11$
 A) No B) Yes
 Answer: A
- 19) $5x_1 + 2x_2 + x_3 = -11$ 19) _____
 $2x_1 - 3x_2 - x_3 = 17$
 $7x_1 - x_2 = 12$
 A) Yes B) No
 Answer: B
- 20) $5x_2 + x_4 = -11$ 20) _____
 $x_1 + x_2 + 6x_3 - x_4 = 15$
 $5x_1 + x_3 + 6x_4 = 16$
 $x_1 + x_2 + 3x_3 = 8$
 A) Yes B) No
 Answer: A

Determine whether the matrix is in echelon form, reduced echelon form, or neither.

$$21) \begin{bmatrix} 1 & 2 & 5 & -7 \\ 0 & 1 & -4 & 9 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

21) _____

- A) Reduced echelon form B) Echelon form C) Neither
Answer: B

$$22) \begin{bmatrix} 1 & 4 & 5 & -7 \\ 0 & 1 & -4 & -6 \\ 0 & 2 & 1 & 6 \end{bmatrix}$$

22) _____

- A) Echelon form B) Reduced echelon form C) Neither
Answer: C

$$23) \begin{bmatrix} 1 & 4 & 5 & -7 \\ 6 & 1 & -4 & 8 \\ 0 & 5 & 1 & 6 \end{bmatrix}$$

23) _____

- A) Reduced echelon form B) Echelon form C) Neither
Answer: C

$$24) \begin{bmatrix} 1 & 0 & 0 & -7 \\ 2 & 1 & 0 & -2 \\ 0 & 5 & 1 & 2 \end{bmatrix}$$

24) _____

- A) Neither B) Echelon form C) Reduced echelon form
Answer: A

$$25) \begin{bmatrix} 1 & 6 & 2 & -7 \\ 0 & 1 & -4 & -6 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

25) _____

- A) Echelon form B) Neither C) Reduced echelon form
Answer: A

$$26) \begin{bmatrix} 1 & 0 & -3 & -5 \\ 0 & 1 & -3 & 4 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

26) _____

- A) Echelon form B) Reduced echelon form C) Neither
Answer: B

$$27) \begin{bmatrix} 1 & -5 & 3 & 4 \\ 0 & 0 & -5 & -4 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

27) _____

- A) Neither B) Echelon form C) Reduced echelon form
Answer: B

Use the row reduction algorithm to transform the matrix into echelon form or reduced echelon form as indicated.

28) Find the echelon form of the given matrix.

28) _____

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ -3 & -11 & 9 & -5 \\ 2 & 2 & 5 & -1 \end{bmatrix}$$

A)

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 15 & -1 \end{bmatrix}$$

B)

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & -6 & 9 & -7 \end{bmatrix}$$

C)

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 27 & 17 \end{bmatrix}$$

D)

$$\begin{bmatrix} 1 & 4 & -2 & 3 \\ 0 & 1 & 3 & 4 \\ 0 & 0 & 27 & 0 \end{bmatrix}$$

Answer: C

29) Find the reduced echelon form of the given matrix.

29) _____

$$\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 2 & 5 & -4 & -1 & 4 \\ -3 & -9 & 9 & 2 & 10 \end{bmatrix}$$

A)

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 26 \\ 0 & 1 & 0 & 0 & -8 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

B)

$$\begin{bmatrix} 1 & 4 & -5 & 1 & 2 \\ 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

C)

$$\begin{bmatrix} 1 & 4 & -5 & 0 & -6 \\ 0 & 1 & -2 & 0 & -8 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

D)

$$\begin{bmatrix} 1 & 0 & 3 & 0 & 26 \\ 0 & 1 & -2 & 0 & -8 \\ 0 & 0 & 0 & 1 & 8 \end{bmatrix}$$

Answer: D

The augmented matrix is given for a system of equations. If the system is consistent, find the general solution.

Otherwise state that there is no solution.

30) $\begin{bmatrix} 1 & -5 & -1 \\ 0 & 0 & 7 \end{bmatrix}$

30) _____

A) $(-1, 7)$

B) $x_1 = -1 + 5x_2$
 x_2 is free

C) No solution

D) $x_1 = -1 + 5x_2$
 $x_2 = 7$
 x_3 is free

Answer: C

31) $\begin{bmatrix} 1 & 2 & -3 & -6 \\ 0 & 1 & 4 & 7 \\ 0 & 0 & 0 & 2 \end{bmatrix}$

31) _____

A) $x_1 = -20 + 11x_3$
 $x_2 = 7 - 4x_3$
 $x_3 = 2$

B) No solution

C) $x_1 = -6 - 2x_2 + 3x_3$
 x_2 is free
 x_3 is free

D) $x_1 = -20 + 11x_3$
 $x_2 = 7 - 4x_3$
 x_3 is free

Answer: B

$$32) \begin{bmatrix} 1 & 2 & -3 & 5 \\ 0 & 1 & 4 & -5 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

32) _____

A) $x_1 = 5 - 2x_2 + 3x_3$

$x_2 = -5 - 4x_3$

x_3 is free

C) $x_1 = 15 + 11x_3$

$x_2 = -5 - 4x_3$

$x_3 = 0$

B) $x_1 = 5 - 2x_2 + 3x_3$

x_2 is free

x_3 is free

D) $x_1 = 15 + 11x_3$

$x_2 = -5 - 4x_3$

x_3 is free

Answer: D

$$33) \begin{bmatrix} 1 & 0 & 6 & 3 \\ 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

33) _____

A) $x_1 = 3 - 6x_3$

$x_2 = -3 + 2x_3$

$x_3 = 0$

B) $x_1 = 3 - 6x_3$

x_2 is free

$x_3 = \frac{3}{2} + \frac{1}{2}x_2$

C) $x_1 = 3 - 6x_3$

$x_2 = -3 + 2x_3$

x_3 is free

D) No solution

Answer: C

$$34) \begin{bmatrix} 1 & 4 & -2 & -3 & 1 \\ 0 & 0 & 1 & 4 & -4 \\ -1 & -4 & 0 & -5 & 7 \end{bmatrix}$$

34) _____

A) $x_1 = -4x_2 + 2x_3 + 3x_4 + 1$

x_2 is free

$x_3 = -4 - 4x_4$

x_4 is free

B) $x_1 = -7 - 4x_2 - 5x_4$

x_2 is free

$x_3 = -4 - 4x_4$

x_4 is free

C) $x_1 = -7 - 4x_2 - 5x_4$

x_2 is free

$x_3 = -4 - 4x_4$

$x_4 = 0$

D) $x_1 = -7 - 4x_2 - 5x_3$

$x_2 = -4 - 4x_3$

x_3 is free

Answer: B

$$35) \begin{bmatrix} 1 & 5 & 8 & -1 & 2 & 5 \\ 0 & 0 & 0 & -4 & 3 & 4 \\ 0 & 0 & 0 & 0 & -2 & 8 \end{bmatrix}$$

35) _____

A) $x_1 = -5x_2 - 8x_3 + 9$

x_2 is free

$x_3 = -4$

$x_4 = \frac{3}{4}x_5 - 1$

$x_5 = -4$

B) No solution

C) $x_1 = -5x_2 - 8x_3 + x_4 - 2x_5 + 5$

x_2 is free

x_3 is free

$x_4 = \frac{3}{4}x_5 - 1$

$x_5 = -4$

D) $x_1 = -5x_2 - 8x_3 + 9$

x_2 is free

x_3 is free

$x_4 = -4$

$x_5 = -4$

Answer: D

Find the indicated vector.

36) Let $\mathbf{u} = \begin{bmatrix} -9 \\ 7 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -3 \\ 4 \end{bmatrix}$. Find $\mathbf{u} + \mathbf{v}$.

36) _____

A) $\begin{bmatrix} -6 \\ 3 \end{bmatrix}$

B) $\begin{bmatrix} -2 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} -5 \\ 4 \end{bmatrix}$

D) $\begin{bmatrix} -12 \\ 11 \end{bmatrix}$

Answer: D

37) Let $\mathbf{u} = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$. Find $\mathbf{u} - \mathbf{v}$.

37) _____

A) $\begin{bmatrix} 3 \\ -2 \end{bmatrix}$

B) $\begin{bmatrix} -2 \\ -7 \end{bmatrix}$

C) $\begin{bmatrix} -5 \\ 4 \end{bmatrix}$

D) $\begin{bmatrix} -4 \\ 5 \end{bmatrix}$

Answer: A

38) Let $\mathbf{u} = \begin{bmatrix} 2 \\ 7 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -8 \\ -5 \end{bmatrix}$. Find $\mathbf{v} - \mathbf{u}$.

38) _____

A) $\begin{bmatrix} -6 \\ 2 \end{bmatrix}$

B) $\begin{bmatrix} 5 \\ 3 \end{bmatrix}$

C) $\begin{bmatrix} -10 \\ -12 \end{bmatrix}$

D) $\begin{bmatrix} -7 \\ -15 \end{bmatrix}$

Answer: C

39) Let $\mathbf{u} = \begin{bmatrix} 6 \\ 1 \end{bmatrix}$. Find $5\mathbf{u}$.

39) _____

A) $\begin{bmatrix} -30 \\ -5 \end{bmatrix}$

B) $\begin{bmatrix} 30 \\ 5 \end{bmatrix}$

C) $\begin{bmatrix} -30 \\ 5 \end{bmatrix}$

D) $\begin{bmatrix} 30 \\ -5 \end{bmatrix}$

Answer: B

40) Let $\mathbf{u} = \begin{bmatrix} 4 \\ -7 \end{bmatrix}$. Find $7\mathbf{u}$.

40) _____

A)

$$\begin{bmatrix} 28 \\ 49 \end{bmatrix}$$

B)

$$\begin{bmatrix} 28 \\ -49 \end{bmatrix}$$

C)

$$\begin{bmatrix} -28 \\ -49 \end{bmatrix}$$

D)

$$\begin{bmatrix} -28 \\ 49 \end{bmatrix}$$

Answer: B

41) Let $\mathbf{u} = \begin{bmatrix} -9 \\ -2 \end{bmatrix}$. Find $-3\mathbf{u}$.

41) _____

A)

$$\begin{bmatrix} 27 \\ 6 \end{bmatrix}$$

B)

$$\begin{bmatrix} -27 \\ 6 \end{bmatrix}$$

C)

$$\begin{bmatrix} -27 \\ -6 \end{bmatrix}$$

D)

$$\begin{bmatrix} 27 \\ -6 \end{bmatrix}$$

Answer: A

42) Let $\mathbf{u} = \begin{bmatrix} 7 \\ -1 \end{bmatrix}$. Find $-6\mathbf{u}$.

42) _____

A)

$$\begin{bmatrix} -42 \\ -6 \end{bmatrix}$$

B)

$$\begin{bmatrix} 42 \\ 6 \end{bmatrix}$$

C)

$$\begin{bmatrix} 42 \\ -6 \end{bmatrix}$$

D)

$$\begin{bmatrix} -42 \\ 6 \end{bmatrix}$$

Answer: D

43) Let $\mathbf{u} = \begin{bmatrix} -2 \\ -9 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 6 \\ -4 \end{bmatrix}$. Find $-2\mathbf{u} + 5\mathbf{v}$.

43) _____

A)

$$\begin{bmatrix} -8 \\ -65 \end{bmatrix}$$

B)

$$\begin{bmatrix} -26 \\ 38 \end{bmatrix}$$

C)

$$\begin{bmatrix} 22 \\ 10 \end{bmatrix}$$

D)

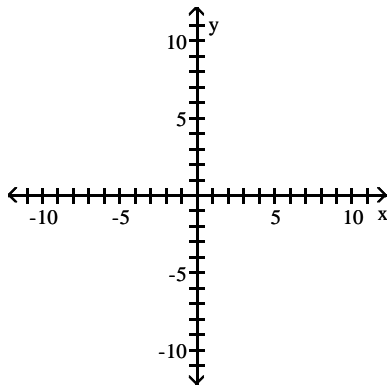
$$\begin{bmatrix} 34 \\ -2 \end{bmatrix}$$

Answer: D

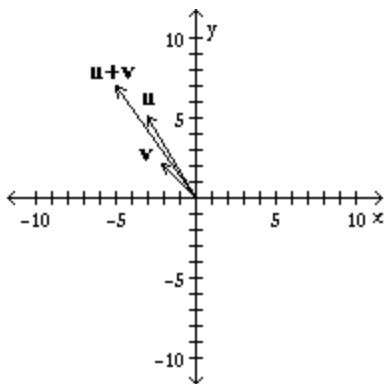
Display the indicated vector(s) on an xy-graph.

44) Let $\mathbf{u} = \begin{bmatrix} -3 \\ 5 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$. Display the vectors \mathbf{u} , \mathbf{v} , and $\mathbf{u} + \mathbf{v}$ on the same axes.

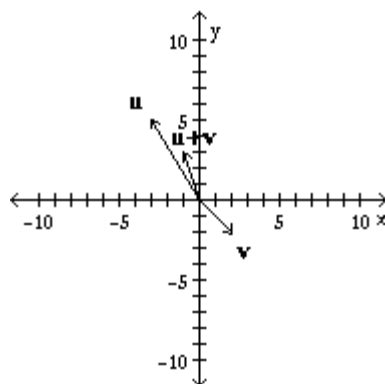
44) _____



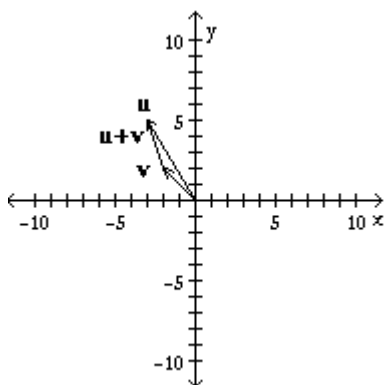
A)



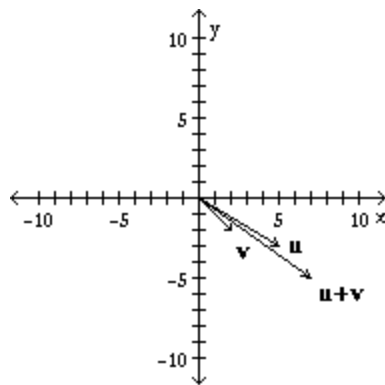
B)



C)



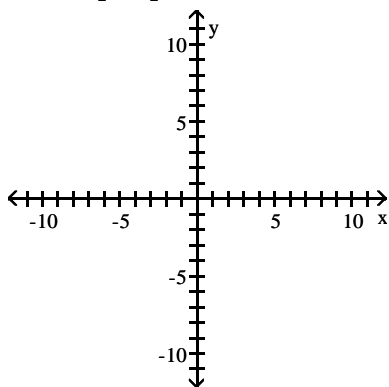
D)



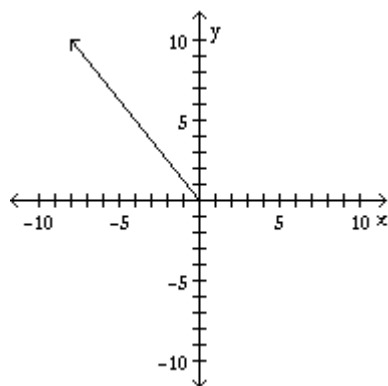
Answer: A

45) Let $\mathbf{u} = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$ Display the vector $2\mathbf{u}$ using the given axes.

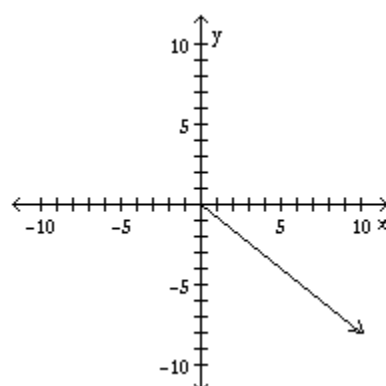
45) _____



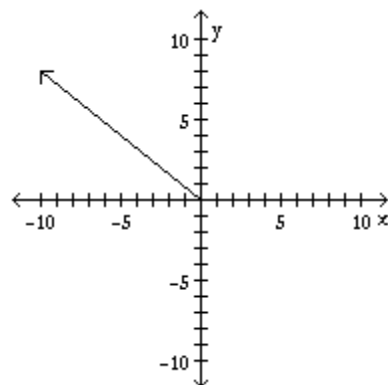
A)



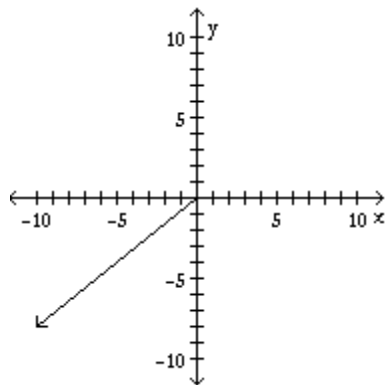
B)



C)



D)



Answer: B

Solve the problem.

46) Let $\mathbf{a}_1 = \begin{bmatrix} 3 \\ 4 \\ -4 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -4 \\ 1 \\ 1 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} 2 \\ -10 \\ 6 \end{bmatrix}$.

46) _____

Determine whether \mathbf{b} can be written as a linear combination of \mathbf{a}_1 and \mathbf{a}_2 . In other words, determine whether weights x_1 and x_2 exist, such that $x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 = \mathbf{b}$. Determine the weights x_1 and x_2 if possible.

A) No solution

B) $x_1 = -1, x_2 = -3$ C) $x_1 = -2, x_2 = -2$ D) $x_1 = -2, x_2 = -1$

Answer: C

47) Let $\mathbf{a}_1 = \begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$, $\mathbf{a}_2 = \begin{bmatrix} -3 \\ -4 \\ 1 \end{bmatrix}$, $\mathbf{a}_3 = \begin{bmatrix} 2 \\ 1 \\ 6 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix}$.

47) _____

Determine whether \mathbf{b} can be written as a linear combination of \mathbf{a}_1 , \mathbf{a}_2 , and \mathbf{a}_3 . In other words, determine whether weights x_1 , x_2 , and x_3 exist, such that $x_1 \mathbf{a}_1 + x_2 \mathbf{a}_2 + x_3 \mathbf{a}_3 = \mathbf{b}$. Determine the weights x_1 , x_2 , and x_3 if possible.

A) $x_1 = 2, x_2 = 1, x_3 = 0$ B) $x_1 = -3, x_2 = 0, x_3 = 1$

C) No solution

D) $x_1 = -2, x_2 = -1, x_3 = 1$

Answer: C

SHORT ANSWER. Write the word or phrase that best completes each statement or answers the question.

- 48) A company manufactures two products. For \$1.00 worth of product A, the company spends \$0.40 on materials, \$0.25 on labor, and \$0.10 on overhead. For \$1.00 worth of product B, the company spends \$0.50 on materials, \$0.20 on labor, and \$0.10 on overhead.
Let

48) _____

$$\mathbf{a} = \begin{bmatrix} 0.40 \\ 0.25 \\ 0.10 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.10 \end{bmatrix}.$$

Then \mathbf{a} and \mathbf{b} represent the "costs per dollar of income" for the two products. Evaluate $100\mathbf{a} + 400\mathbf{b}$ and give an economic interpretation of the result.

$$\text{Answer: } 100\mathbf{a} + 400\mathbf{b} = \begin{bmatrix} 240 \\ 105 \\ 50 \end{bmatrix}$$

$100\mathbf{a} + 400\mathbf{b}$ lists the various costs for producing \$100 worth of product A and \$400 worth of product B, namely \$240 for materials, \$105 for labor, and \$50 for overhead.

- 49) A company manufactures two products. For \$1.00 worth of product A, the company spends \$0.50 on materials, \$0.20 on labor, and \$0.15 on overhead. For \$1.00 worth of product B, the company spends \$0.45 on materials, \$0.20 on labor, and \$0.15 on overhead.
Let

49) _____

$$\mathbf{a} = \begin{bmatrix} 0.50 \\ 0.20 \\ 0.15 \end{bmatrix} \text{ and } \mathbf{b} = \begin{bmatrix} 0.45 \\ 0.20 \\ 0.15 \end{bmatrix}.$$

Then \mathbf{a} and \mathbf{b} represent the "costs per dollar of income" for the two products. Suppose the company manufactures x_1 dollars worth of product A and x_2 dollars worth of product B and that its total costs for materials are \$140, its total costs for labor are \$60, and its total costs for overhead are \$45. Determine x_1 and x_2 , the dollars worth of each product produced. Include a vector equation as part of your solution.

$$\text{Answer: } x_1\mathbf{a} + x_2\mathbf{b} = \begin{bmatrix} 140 \\ 60 \\ 45 \end{bmatrix}$$

or

$$x_1 \begin{bmatrix} 0.50 \\ 0.20 \\ 0.15 \end{bmatrix} + x_2 \begin{bmatrix} 0.45 \\ 0.20 \\ 0.15 \end{bmatrix} = \begin{bmatrix} 140 \\ 60 \\ 45 \end{bmatrix}$$

$$x_1 = 100, x_2 = 200$$

MULTIPLE CHOICE. Choose the one alternative that best completes the statement or answers the question.

Compute the product or state that it is undefined.

50) $[-7 \ 2 \ 7] \begin{bmatrix} 3 \\ 0 \\ -3 \end{bmatrix}$

50) _____

A) $[141]$

B) $\begin{bmatrix} -21 \\ 0 \\ -21 \end{bmatrix}$

C) $[-21 \ 0 \ -21]$

D) $[-42]$

Answer: D

51) $\begin{bmatrix} -2 & -2 & 6 \\ 5 & 8 & -5 \end{bmatrix} \begin{bmatrix} 8 \\ -3 \\ 3 \end{bmatrix}$

51) _____

A) $\begin{bmatrix} 8 \\ 1 \end{bmatrix}$

B) $\begin{bmatrix} 1 \\ 8 \end{bmatrix}$

C) $\begin{bmatrix} -2 & -2 & 6 \\ 5 & 8 & -5 \\ 8 & -3 & 3 \end{bmatrix}$

D) $[8 \ 1]$

Answer: A

52) $\begin{bmatrix} -1 & 3 \\ -8 & -5 \\ -6 & -8 \end{bmatrix} \begin{bmatrix} 3 \\ -1 \end{bmatrix}$

52) _____

A) $\begin{bmatrix} -6 \\ -19 \\ -10 \end{bmatrix}$

B) Undefined

C) $\begin{bmatrix} -3 & -3 \\ -24 & 5 \\ -18 & 8 \end{bmatrix}$

D) $\begin{bmatrix} -3 & 8 \\ 9 & 5 \\ -6 & -8 \end{bmatrix}$

Answer: A

53) $\begin{bmatrix} 5 & -3 \\ -3 & 4 \\ -3 & 8 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \\ 8 \end{bmatrix}$

53) _____

A) Undefined

B) $\begin{bmatrix} 4 \\ -4 \\ 40 \end{bmatrix}$

C) $[-2 \ 42]$

D) $\begin{bmatrix} 10 & -6 \\ 12 & -16 \\ -24 & 64 \end{bmatrix}$

Answer: A

Write the system as a vector equation or matrix equation as indicated.

54) Write the following system as a vector equation involving a linear combination of vectors. 54) _____

$$6x_1 - 6x_2 - x_3 = 5$$

$$6x_1 + 3x_3 = -5$$

A) $x_1 \begin{bmatrix} 6 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

B) $x_1 \begin{bmatrix} 6 \\ 6 \\ -1 \end{bmatrix} + x_2 \begin{bmatrix} 6 \\ 0 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$

C) $6 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - 6 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} - \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \\ 0 \end{bmatrix}$

D) $x_1 \begin{bmatrix} 6 \\ 6 \end{bmatrix} + x_2 \begin{bmatrix} -6 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 5 \\ -5 \end{bmatrix}$

Answer: A

55) Write the following system as a matrix equation involving the product of a matrix and a vector on the left side and a vector on the right side. 55) _____

$$2x_1 + x_2 - 6x_3 = -6$$

$$6x_1 - 4x_2 = 2$$

A) $\begin{bmatrix} 2 & 1 & -6 \\ 6 & 4 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

B) $\begin{bmatrix} 2 & 1 & -6 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

C) $\begin{bmatrix} 2 & 6 \\ 1 & -4 \\ -6 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

D) $\begin{bmatrix} x_1 & x_2 & x_3 \\ 6 & -4 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ -6 \end{bmatrix} = \begin{bmatrix} -6 \\ 2 \end{bmatrix}$

Answer: B

Solve the problem.

56) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -4 & 5 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. 56) _____

Determine if the equation $Ax = b$ is consistent for all possible b_1, b_2, b_3 . If the equation is not consistent for all possible b_1, b_2, b_3 , give a description of the set of all \mathbf{b} for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

A) Equation is consistent for all b_1, b_2, b_3 satisfying $-3b_1 + b_3 = 0$.

B) Equation is consistent for all b_1, b_2, b_3 satisfying $2b_1 + b_2 = 0$.

C) Equation is consistent for all possible b_1, b_2, b_3 .

D) Equation is consistent for all b_1, b_2, b_3 satisfying $7b_1 + 5b_2 + b_3 = 0$.

Answer: C

57) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -2 & 5 & -1 \\ 3 & -6 & -3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$.

57) _____

Determine if the equation $Ax = b$ is consistent for all possible b_1, b_2, b_3 . If the equation is not consistent for all possible b_1, b_2, b_3 , give a description of the set of all \mathbf{b} for which the equation is consistent (i.e., a condition which must be satisfied by b_1, b_2, b_3).

- A) Equation is consistent for all b_1, b_2, b_3 satisfying $-b_1 + b_2 + b_3 = 0$.
- B) Equation is consistent for all b_1, b_2, b_3 satisfying $-3b_1 + b_3 = 0$.
- C) Equation is consistent for all b_1, b_2, b_3 satisfying $3b_1 + 3b_2 + b_3 = 0$.
- D) Equation is consistent for all possible b_1, b_2, b_3 .

Answer: C

58) Find the general solution of the simple homogeneous "system" below, which consists of a single linear equation. Give your answer as a linear combination of vectors. Let x_2 and x_3 be free variables.

58) _____

$$2x_1 - 16x_2 + 10x_3 = 0$$

A)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 8 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + 5 \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad (\text{with } x_2, x_3 \text{ free})$$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 8 \\ 0 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 1 \\ 0 \end{bmatrix} \quad (\text{with } x_2, x_3 \text{ free})$$

C)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} 8 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 0 \\ 1 \end{bmatrix} \quad (\text{with } x_2, x_3 \text{ free})$$

D)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_2 \begin{bmatrix} -8 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 5 \\ 0 \\ 1 \end{bmatrix} \quad (\text{with } x_2, x_3 \text{ free})$$

Answer: C

59) Find the general solution of the homogeneous system below. Give your answer as a vector.

59) _____

$$x_1 + 2x_2 - 3x_3 = 0$$

$$4x_1 + 7x_2 - 9x_3 = 0$$

$$-x_1 - 4x_2 + 9x_3 = 0$$

A)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} 3 \\ -3 \\ 1 \end{bmatrix}$$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

C)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 3 \\ 1 \end{bmatrix}$$

D)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_3 \begin{bmatrix} -3 \\ 3 \\ 0 \end{bmatrix}$$

Answer: C

60) Describe all solutions of $Ax = b$, where

60) _____

$$A = \begin{bmatrix} 2 & -5 & 3 \\ -2 & 6 & -5 \\ -4 & 7 & 0 \end{bmatrix} \text{ and } b = \begin{bmatrix} -3 \\ 4 \\ 3 \end{bmatrix}.$$

Describe the general solution in parametric vector form.

A)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 7/2 \\ 2 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$$

B)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 7/2 \\ 2 \\ 0 \end{bmatrix}$$

C)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 7/2 \\ 2 \\ 1 \end{bmatrix}$$

D)

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -3 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$$

Answer: C

61) Suppose an economy consists of three sectors: Energy (E), Manufacturing (M), and Agriculture (A).

61) _____

Sector E sells 70% of its output to M and 30% to A.

Sector M sells 30% of its output to E, 50% to A, and retains the rest.

Sector A sells 15% of its output to E, 30% to M, and retains the rest.

Denote the prices (dollar values) of the total annual outputs of the Energy, Manufacturing, and Agriculture sectors by p_e , p_m , and p_a , respectively. If possible, find equilibrium prices that make each sector's income match its expenditures.

Find the general solution as a vector, with p_a free.

A)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.465 p_a \\ 0.593 p_a \\ p_a \end{bmatrix}$$

B)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.356 p_a \\ 0.686 p_a \\ p_a \end{bmatrix}$$

C)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.607 p_a \\ 0.481 p_a \\ p_a \end{bmatrix}$$

D)

$$\begin{bmatrix} p_e \\ p_m \\ p_a \end{bmatrix} = \begin{bmatrix} 0.308 p_a \\ 0.716 p_a \\ p_a \end{bmatrix}$$

Answer: B

66) For what values of h are the given vectors linearly dependent?

66) _____

$$\begin{bmatrix} -1 \\ 4 \\ 6 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ -3 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ h \end{bmatrix}$$

- A) Vectors are linearly dependent for $h = -2$
 C) Vectors are linearly dependent for $h \neq -2$

- B) Vectors are linearly independent for all h
 D) Vectors are linearly dependent for all h

Answer: D

67) Let $A = \begin{bmatrix} 2 & 3 & 1 \\ 8 & -7 & 5 \end{bmatrix}$ and $\mathbf{u} = \begin{bmatrix} 3 \\ 7 \\ 2 \end{bmatrix}$.

67) _____

Define a transformation $T: \mathcal{R}^3 \rightarrow \mathcal{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$. Find $T(\mathbf{u})$, the image of \mathbf{u} under the transformation T .

A)

$$\begin{bmatrix} 29 \\ -15 \end{bmatrix}$$

B)

$$\begin{bmatrix} 6 & 21 & 2 \\ 24 & -49 & 10 \end{bmatrix}$$

C)

$$\begin{bmatrix} 18 \\ 42 \end{bmatrix}$$

D)

$$\begin{bmatrix} 30 \\ -28 \\ 12 \end{bmatrix}$$

Answer: A

68) Let $T: \mathcal{R}^2 \rightarrow \mathcal{R}^2$ be a linear transformation that maps $\mathbf{u} = \begin{bmatrix} -6 \\ 4 \end{bmatrix}$ into $\begin{bmatrix} -22 \\ 12 \end{bmatrix}$ and maps $\mathbf{v} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$ into

68) _____

$$\begin{bmatrix} 11 \\ -4 \end{bmatrix}.$$

Use the fact that T is linear to find the image of $3\mathbf{u} + \mathbf{v}$.

A)

$$\begin{bmatrix} -33 \\ 24 \end{bmatrix}$$

B)

$$\begin{bmatrix} -11 \\ 8 \end{bmatrix}$$

C)

$$\begin{bmatrix} -55 \\ 32 \end{bmatrix}$$

D)

$$\begin{bmatrix} -16 \\ 7 \end{bmatrix}$$

Answer: C

69) Let $A = \begin{bmatrix} 1 & -3 & 0 \\ -4 & 0 & 2 \\ 4 & 1 & -2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 9 \\ -6 \\ 4 \end{bmatrix}$.

69) _____

Define a transformation $T: \mathcal{R}^3 \rightarrow \mathcal{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$.

If possible, find a vector \mathbf{x} whose image under T is \mathbf{b} . Otherwise, state that \mathbf{b} is not in the range of the transformation T .

A)

$$\begin{bmatrix} 3 \\ -2 \\ 0 \end{bmatrix}$$

B)

\mathbf{b} is not in the range of the transformation T .

C)

$$\begin{bmatrix} 3 \\ 3 \\ -2 \end{bmatrix}$$

D)

$$\begin{bmatrix} 3 \\ -2 \\ 3 \end{bmatrix}$$

Answer: D

70) Let $A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 4 & -1 \\ 2 & -5 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 2 \\ 4 \\ -4 \end{bmatrix}$.

70) _____

Define a transformation $T: \mathcal{R}^3 \rightarrow \mathcal{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$.

If possible, find a vector \mathbf{x} whose image under T is \mathbf{b} . Otherwise, state that \mathbf{b} is not in the range of the transformation T .

A)

$$\begin{bmatrix} 4 \\ 2 \\ 2 \end{bmatrix}$$

B)

$$\begin{bmatrix} 4 \\ 4 \\ 0 \end{bmatrix}$$

C)

$$\begin{bmatrix} 4 \\ 0 \\ -4 \end{bmatrix}$$

D)

\mathbf{b} is not in the range of the transformation T .

Answer: D

Describe geometrically the effect of the transformation T .

71) Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$.

71) _____

Define a transformation T by $T(\mathbf{x}) = A\mathbf{x}$.

A) Vertical shear

B) Projection onto x_2 -axis

C) Projection onto x_1 -axis

D) Horizontal shear

Answer: A

72) Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$.

72) _____

Define a transformation T by $T(\mathbf{x}) = A\mathbf{x}$.

A) Projection onto the x_2x_3 -plane

B) Projection onto the x_2 -axis

C) Horizontal shear

D) Vertical shear

Answer: A

Solve the problem.

73) The columns of $I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ are $\mathbf{e}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\mathbf{e}_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$, $\mathbf{e}_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$.

73) _____

Suppose that T is a linear transformation from \mathcal{R}^3 into \mathcal{R}^2 such that

$$T(\mathbf{e}_1) = \begin{bmatrix} 3 \\ -2 \end{bmatrix}, T(\mathbf{e}_2) = \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \text{ and } T(\mathbf{e}_3) = \begin{bmatrix} -5 \\ 1 \end{bmatrix}.$$

Find a formula for the image of an arbitrary $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ in \mathcal{R}^3 .

A)

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_2 \\ 5x_1 \end{bmatrix}$$

B)

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 - 2x_2 \\ 5x_1 \\ 5x_2 + x_3 \end{bmatrix}$$

C)

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 5x_2 - 5x_3 \\ 5x_1 \\ -2x_1 + x_3 \end{bmatrix}$$

D)

$$T \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3x_1 + 5x_2 - 5x_3 \\ -2x_1 + x_3 \end{bmatrix}$$

Answer: D

Find the standard matrix of the linear transformation T .

74) $T: \mathcal{R}^2 \rightarrow \mathcal{R}^2$ rotates points (about the origin) through $\frac{7}{4}\pi$ radians (with counterclockwise rotation for a positive angle).

74) _____

A)

$$\begin{bmatrix} \frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \\ -\frac{\sqrt{3}}{3} & \frac{\sqrt{3}}{3} \end{bmatrix}$$

B)

$$\begin{bmatrix} -\frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

C)

$$\begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$$

D)

$$\begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix}$$

Answer: C

75) $T: \mathcal{R}^2 \rightarrow \mathcal{R}^2$ first performs a vertical shear that maps \mathbf{e}_1 into $\mathbf{e}_1 + 3\mathbf{e}_2$, but leaves the vector \mathbf{e}_2 unchanged, then reflects the result through the horizontal x_1 -axis.

75) _____

A)

$$\begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}$$

B)

$$\begin{bmatrix} 1 & 3 \\ 0 & -1 \end{bmatrix}$$

C)

$$\begin{bmatrix} 1 & 0 \\ -3 & -1 \end{bmatrix}$$

D)

$$\begin{bmatrix} -1 & 0 \\ 3 & -1 \end{bmatrix}$$

Answer: C

