

# INSTRUCTOR'S SOLUTIONS MANUAL

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*University of California, Davis*

## THOMAS' CALCULUS EARLY TRANSCENDENTALS FOURTEENTH EDITION

*Based on the original work by*

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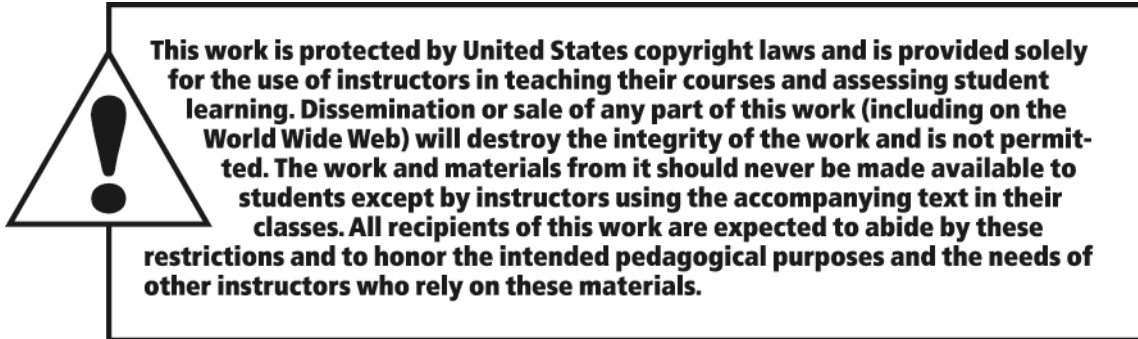
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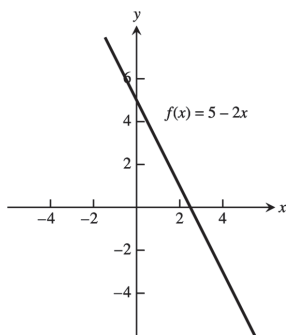
# CHAPTER 1 FUNCTIONS

## 1.1 FUNCTIONS AND THEIR GRAPHS

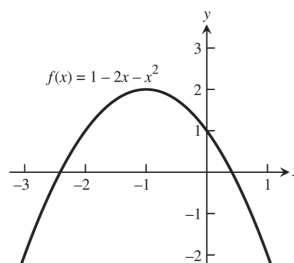
1. domain =  $(-\infty, \infty)$ ; range =  $[1, \infty)$
2. domain =  $[0, \infty)$ ; range =  $(-\infty, 1]$
3. domain =  $[-2, \infty)$ ;  $y$  in range and  $y = \sqrt{5x+10} \geq 0 \Rightarrow y$  can be any positive real number  $\Rightarrow$  range =  $[0, \infty)$ .
4. domain =  $(-\infty, 0] \cup [3, \infty)$ ;  $y$  in range and  $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow y$  can be any positive real number  $\Rightarrow$  range =  $[0, \infty)$ .
5. domain =  $(-\infty, 3) \cup (3, \infty)$ ;  $y$  in range and  $y = \frac{4}{3-t}$ , now if  $t < 3 \Rightarrow 3-t > 0 \Rightarrow \frac{4}{3-t} > 0$ , or if  $t > 3 \Rightarrow 3-t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow y$  can be any nonzero real number  $\Rightarrow$  range =  $(-\infty, 0) \cup (0, \infty)$ .
6. domain =  $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$ ;  $y$  in range and  $y = \frac{2}{t^2-16}$ , now if  $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$ , or if  $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$ , or if  $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow y$  can be any nonzero real number  $\Rightarrow$  range =  $(-\infty, -\frac{1}{8}] \cup (0, \infty)$ .
7. (a) Not the graph of a function of  $x$  since it fails the vertical line test.  
(b) Is the graph of a function of  $x$  since any vertical line intersects the graph at most once.
8. (a) Not the graph of a function of  $x$  since it fails the vertical line test.  
(b) Not the graph of a function of  $x$  since it fails the vertical line test.
9. base =  $x$ ; (height) $^2 + (\frac{x}{2})^2 = x^2 \Rightarrow$  height =  $\frac{\sqrt{3}}{2}x$ ; area is  $a(x) = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$ ;  
perimeter is  $p(x) = x + x + x = 3x$ .
10.  $s =$  side length  $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$ ; and area is  $a = s^2 \Rightarrow a = \frac{1}{2}d^2$
11. Let  $D =$  diagonal length of a face of the cube and  $\ell =$  the length of an edge. Then  $\ell^2 + D^2 = d^2$  and  $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$ . The surface area is  $6\ell^2 = \frac{6d^2}{3} = 2d^2$  and the volume is  $\ell^3 = \left(\frac{d}{\sqrt{3}}\right)^3 = \frac{d^3}{3\sqrt{3}}$ .
12. The coordinates of  $P$  are  $(x, \sqrt{x})$  so the slope of the line joining  $P$  to the origin is  $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} (x > 0)$ .  
Thus,  $(x, \sqrt{x}) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$ .
13.  $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}$ ;  $L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + \left(-\frac{1}{2}x + \frac{5}{4}\right)^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$   
 $= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$
14.  $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x$ ;  $L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$   
 $= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

2 Chapter 1 Functions

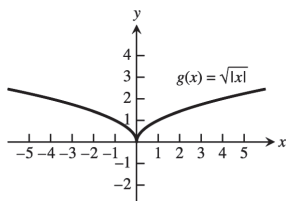
15. The domain is  $(-\infty, \infty)$ .



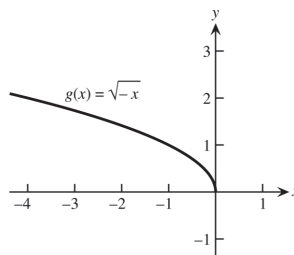
16. The domain is  $(-\infty, \infty)$ .



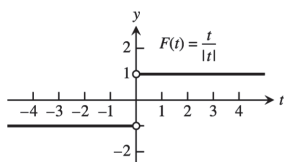
17. The domain is  $(-\infty, \infty)$ .



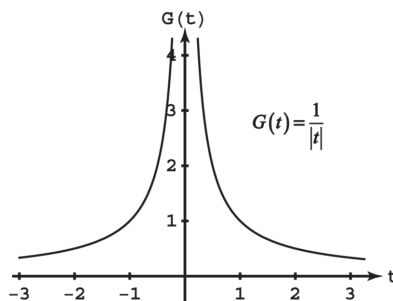
18. The domain is  $(-\infty, 0]$ .



19. The domain is  $(-\infty, 0) \cup (0, \infty)$ .



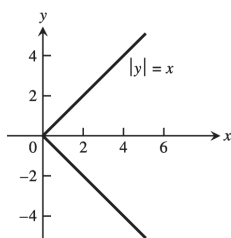
20. The domain is  $(-\infty, 0) \cup (0, \infty)$ .



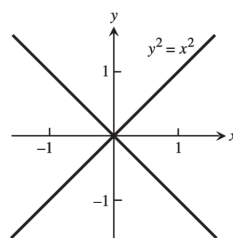
21. The domain is  $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$  22. The range is  $[5, \infty)$ .

23. Neither graph passes the vertical line test

(a)

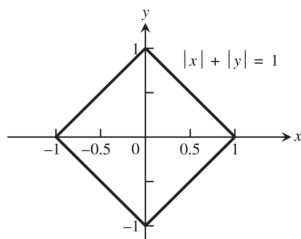


(b)

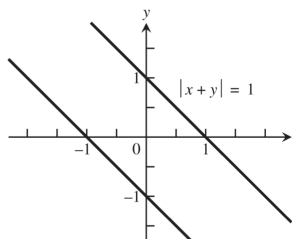


24. Neither graph passes the vertical line test

(a)



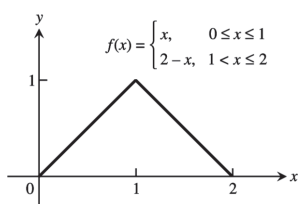
(b)



$$|x+y|=1 \Leftrightarrow \begin{cases} x+y=1 \\ \text{or} \\ x+y=-1 \end{cases} \Leftrightarrow \begin{cases} y=1-x \\ \text{or} \\ y=-1-x \end{cases}$$

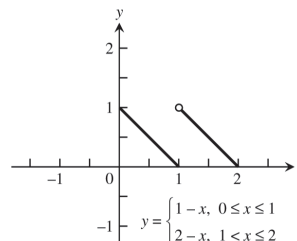
25.

x	0	1	2
y	0	1	0

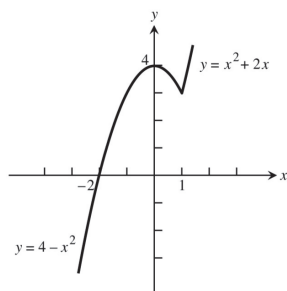


26.

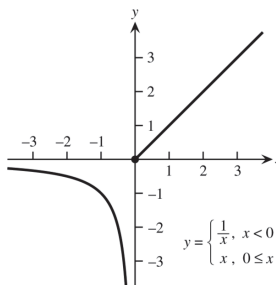
x	0	1	2
y	1	0	0



27.  $F(x) = \begin{cases} 4-x^2, & x \leq 1 \\ x^2+2x, & x > 1 \end{cases}$



28.  $G(x) = \begin{cases} \frac{1}{x}, & x < 0 \\ x, & 0 \leq x \end{cases}$



29. (a) Line through (0, 0) and (1, 1):  $y = x$ ; Line through (1, 1) and (2, 0):  $y = -x + 2$

$$f(x) = \begin{cases} x, & 0 \leq x \leq 1 \\ -x+2, & 1 < x \leq 2 \end{cases}$$

(b)  $f(x) = \begin{cases} 2, & 0 \leq x < 1 \\ 0, & 1 \leq x < 2 \\ 2, & 2 \leq x < 3 \\ 0, & 3 \leq x \leq 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0):  $y = -x + 2$

Line through (2, 1) and (5, 0):  $m = \frac{0-1}{5-2} = -\frac{1}{3}$ , so  $y = -\frac{1}{3}(x-2) + 1 = -\frac{1}{3}x + \frac{5}{3}$

$$f(x) = \begin{cases} -x+2, & 0 < x \leq 2 \\ -\frac{1}{3}x + \frac{5}{3}, & 2 < x \leq 5 \end{cases}$$

4 Chapter 1 Functions

- (b) Line through  $(-1, 0)$  and  $(0, -3)$ :  $m = \frac{-3-0}{0-(-1)} = -3$ , so  $y = -3x - 3$   
 Line through  $(0, 3)$  and  $(2, -1)$ :  $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$ , so  $y = -2x + 3$

$$f(x) = \begin{cases} -3x-3, & -1 < x \leq 0 \\ -2x+3, & 0 < x \leq 2 \end{cases}$$

31. (a) Line through  $(-1, 1)$  and  $(0, 0)$ :  $y = -x$   
 Line through  $(0, 1)$  and  $(1, 1)$ :  $y = 1$   
 Line through  $(1, 1)$  and  $(3, 0)$ :  $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$ , so  $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \leq x < 0 \\ 1 & 0 < x \leq 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

- (b) Line through  $(-2, -1)$  and  $(0, 0)$ :  $y = \frac{1}{2}x$   
 Line through  $(0, 2)$  and  $(1, 0)$ :  $y = -2x + 2$   
 Line through  $(1, -1)$  and  $(3, -1)$ :  $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \leq x \leq 0 \\ -2x+2 & 0 < x \leq 1 \\ -1 & 1 < x \leq 3 \end{cases}$$

32. (a) Line through  $(\frac{T}{2}, 0)$  and  $(T, 1)$ :  $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$ , so  $y = \frac{2}{T}(x - \frac{T}{2}) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \leq x \leq \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \leq T \end{cases}$$

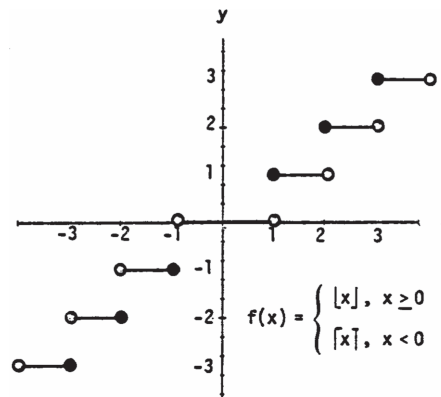
- (b)  $f(x) = \begin{cases} A, & 0 \leq x < \frac{T}{2} \\ -A, & \frac{T}{2} \leq x < T \\ A, & T \leq x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \leq x \leq 2T \end{cases}$

33. (a)  $\lfloor x \rfloor = 0$  for  $x \in [0, 1)$  (b)  $\lceil x \rceil = 0$  for  $x \in (-1, 0]$

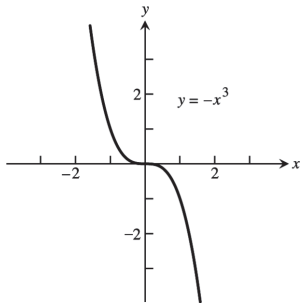
34.  $\lfloor x \rfloor = \lceil x \rceil$  only when  $x$  is an integer.

35. For any real number  $x$ ,  $n \leq x \leq n+1$ , where  $n$  is an integer. Now:  $n \leq x \leq n+1 \Rightarrow -(n+1) \leq -x \leq -n$ .  
 By definition:  $\lceil -x \rceil = -n$  and  $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$ . So  $\lceil -x \rceil = -\lfloor x \rfloor$  for all real  $x$ .

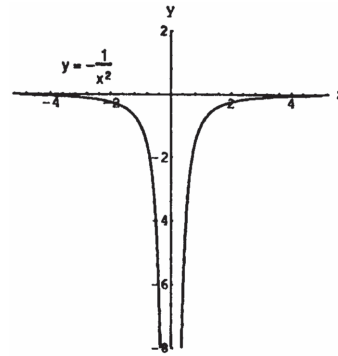
36. To find  $f(x)$  you delete the decimal or fractional portion of  $x$ , leaving only the integer part.



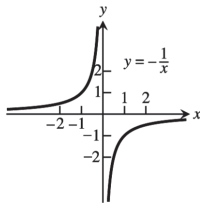
37. Symmetric about the origin  
Dec:  $-\infty < x < \infty$   
Inc: nowhere



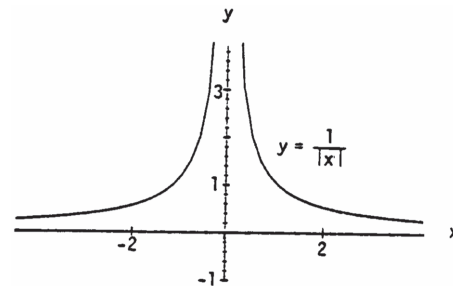
38. Symmetric about the y-axis  
Dec:  $-\infty < x < 0$   
Inc:  $0 < x < \infty$



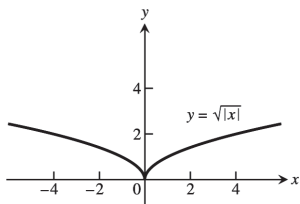
39. Symmetric about the origin  
Dec: nowhere  
Inc:  $-\infty < x < 0$   
 $0 < x < \infty$



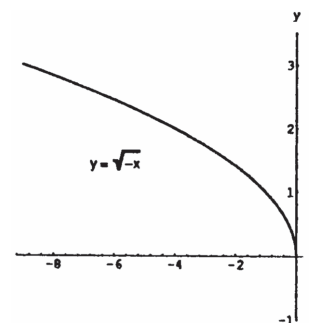
40. Symmetric about the y-axis  
Dec:  $0 < x < \infty$   
Inc:  $-\infty < x < 0$



41. Symmetric about the y-axis  
Dec:  $-\infty < x \leq 0$   
Inc:  $0 \leq x < \infty$

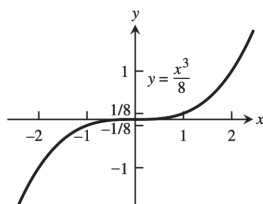


42. No symmetry  
Dec:  $-\infty < x \leq 0$   
Inc: nowhere

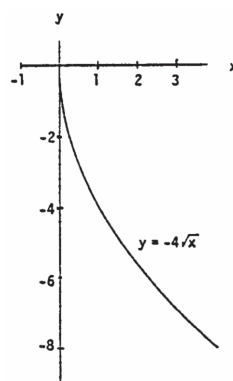


6 Chapter 1 Functions

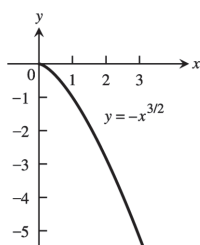
43. Symmetric about the origin  
Dec: nowhere  
Inc:  $-\infty < x < \infty$



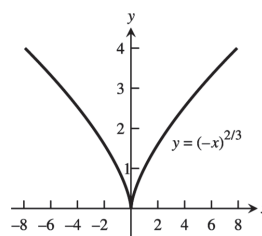
44. No symmetry  
Dec:  $0 \leq x < \infty$   
Inc: nowhere



45. No symmetry  
Dec:  $0 \leq x < \infty$   
Inc: nowhere



46. Symmetric about the y-axis  
Dec:  $-\infty < x \leq 0$   
Inc:  $0 \leq x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48.  $f(x) = x^{-5} = \frac{1}{x^5}$  and  $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$ . Thus the function is odd.

49. Since  $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$ . The function is even.

50. Since  $[f(x) = x^2 + x] \neq [f(-x) = (-x)^2 - x]$  and  $[f(x) = x^2 + x] \neq [-f(x) = -(x)^2 - x]$  the function is neither even nor odd.

51. Since  $g(x) = x^3 + x$ ,  $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$ . So the function is odd.

52.  $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$ , thus the function is even.

53.  $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$ . Thus the function is even.

54.  $g(x) = \frac{x}{x^2 - 1}$ ;  $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$ . So the function is odd.

55.  $h(t) = \frac{1}{t - 1}$ ;  $h(-t) = \frac{1}{-t - 1}$ ;  $-h(t) = \frac{1}{1 - t}$ . Since  $h(t) \neq -h(t)$  and  $h(t) \neq h(-t)$ , the function is neither even nor odd.

56. Since  $|t^3| = |(-t)^3|$ ,  $h(t) = h(-t)$  and the function is even.
57.  $h(t) = 2t + 1$ ,  $h(-t) = -2t + 1$ . So  $h(t) \neq h(-t)$ .  $-h(t) = -2t - 1$ , so  $h(t) \neq -h(t)$ . The function is neither even nor odd.
58.  $h(t) = 2|t| + 1$  and  $h(-t) = 2|-t| + 1 = 2|t| + 1$ . So  $h(t) = h(-t)$  and the function is even.
59.  $g(x) = \sin 2x$ ;  $g(-x) = -\sin 2x = -g(x)$ . So the function is odd.
60.  $g(x) = \sin x^2$ ;  $g(-x) = \sin x^2 = g(x)$ . So the function is even.
61.  $g(x) = \cos 3x$ ;  $g(-x) = \cos 3x = g(x)$ . So the function is even.
62.  $g(x) = 1 + \cos x$ ;  $g(-x) = 1 + \cos x = g(x)$ . So the function is even.
63.  $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t$ ;  $60 = \frac{1}{3}t \Rightarrow t = 180$
64.  $K = cv^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2$ ;  $K = 40(10)^2 = 4000$  joules
65.  $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}$ ;  $10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$
66.  $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}$ ;  $23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2 \text{ in}^3$
67.  $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x$ ;  $0 < x < 7$ .
68. (a) Let  $h$  = height of the triangle. Since the triangle is isosceles,  $(\overline{AB})^2 + (\overline{AB})^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$ . So,  
 $h^2 + 1^2 = (\sqrt{2})^2 \Rightarrow h = 1 \Rightarrow B$  is at  $(0, 1) \Rightarrow$  slope of  $AB = -1 \Rightarrow$  The equation of  $AB$  is  
 $y = f(x) = -x + 1$ ;  $x \in [0, 1]$ .
- (b)  $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x$ ;  $x \in [0, 1]$ .
69. (a) Graph  $h$  because it is an even function and rises less rapidly than does Graph  $g$ .  
 (b) Graph  $f$  because it is an odd function.  
 (c) Graph  $g$  because it is an even function and rises more rapidly than does Graph  $h$ .
70. (a) Graph  $f$  because it is linear.  
 (b) Graph  $g$  because it contains  $(0, 1)$ .  
 (c) Graph  $h$  because it is a nonlinear odd function.

71. (a) From the graph,  $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

(b)  $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

$$x > 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$$

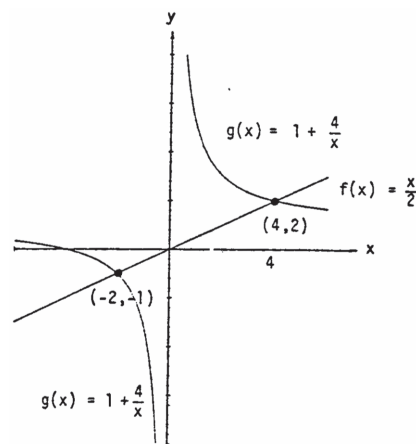
$\Rightarrow x > 4$  since  $x$  is positive;

$$x < 0: \frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$$

$\Rightarrow x < -2$  since  $x$  is negative;  
sign of  $(x-4)(x+2)$



Solution interval:  $(-2, 0) \cup (4, \infty)$



72. (a) From the graph,  $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case  $x < -1$ :  $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$

$$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5.$$

Thus,  $x \in (-\infty, -5)$  solves the inequality.

Case  $-1 < x < 1$ :  $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

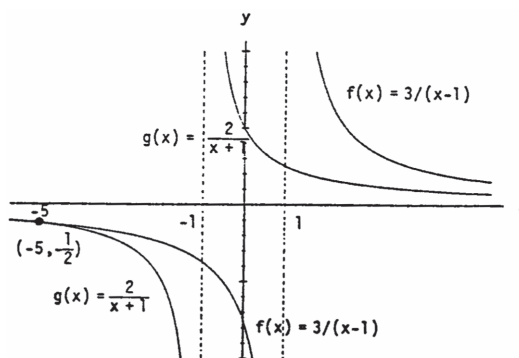
$$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5$$

which is true if  $x > -1$ . Thus,  $x \in (-1, 1)$   
solves the inequality.

Case  $1 < x$ :  $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$

which is never true if  $1 < x$ ,  
so no solution here.

In conclusion,  $x \in (-\infty, -5) \cup (-1, 1)$ .



73. A curve symmetric about the  $x$ -axis will not pass the vertical line test because the points  $(x, y)$  and  $(x, -y)$  lie on the same vertical line. The graph of the function  $y = f(x) = 0$  is the  $x$ -axis, a horizontal line for which there is a single  $y$ -value, 0, for any  $x$ .

74. price =  $40 + 5x$ , quantity =  $300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

75.  $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$ ; cost =  $5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h(\sqrt{2} + 2)$

76. (a) Note that 2 mi = 10,560 ft, so there are  $\sqrt{800^2 + x^2}$  feet of river cable at \$180 per foot and  $(10,560 - x)$  feet of land cable at \$100 per foot. The cost is  $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$ .

(b)  $C(0) = \$1,200,000$

$$C(500) \approx \$1,175,812$$

$$C(1000) \approx \$1,186,512$$

$$C(1500) \approx \$1,212,000$$

$$C(2000) \approx \$1,243,732$$

$$C(2500) \approx \$1,278,479$$

$$C(3000) \approx \$1,314,870$$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point  $P$ .