1. The countries of Oceania and Eurasia are at war.[5] As depicted in the figure, Oceania has four cities—Argula, Betra, Carnat, and Dussel—and it is concerned that one of them is to be bombed by Eurasia. The bombers could come from either base Alpha, which can reach the cities of Argula and Betra; or from base Beta, which can reach either Carnat or Dussel. Eurasia decides which one of these four cities to attack. Oceania doesn't know which one has been selected, but does observe the base from which the bombers are flying. After making that observation, Oceania decides which one (and only one) of its four cities to evacuate. Assign a payoff of 2 to Oceania if it succeeds in evacuating the city that is to be bombed and a payoff of 1 otherwise. Assign Eurasia a payoff of 1 if the city it bombs was not evacuated and a zero payoff otherwise. Write down the extensive form game.
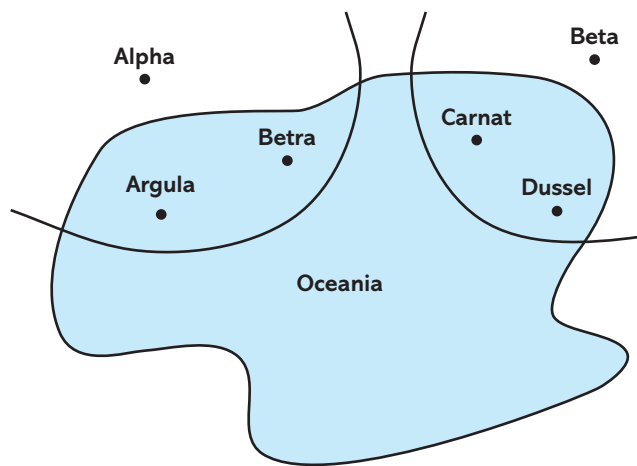

ANSWER: In the figure below, *A* stands for Argula, *B* for Betra, *C* for Carnat, and *D* for Dussel.

|	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D
Eurasia	0	1	1	1	1	0	1	1	1	1	0	1	1	1	1	0
Oceania	2	1	1	1	1	2	1	1	1	1	2	1	1	1	1	2
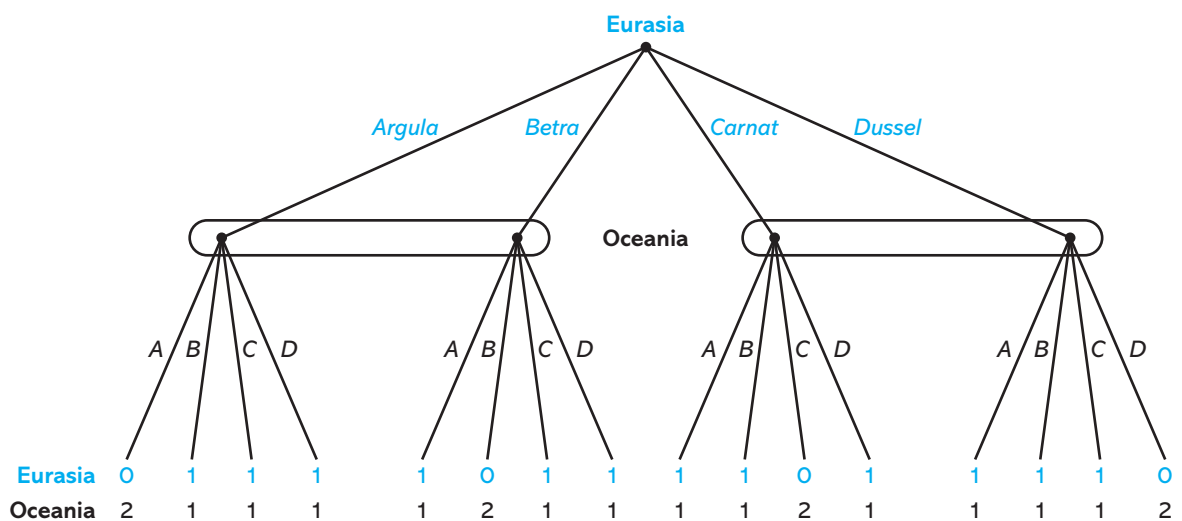
2. Player 1 moves initially by choosing among four actions: *a*, *b*, *c*, and *d*. If player 1 chose anything but *d*, then player 2 chooses between *x* and *y*. Player 2 gets to observe the choice of player 1. If player 1 chose *d*, then player 3 moves by choosing between *left* and *right*. Write down the extensive form of this setting. (You can ignore payoffs.)
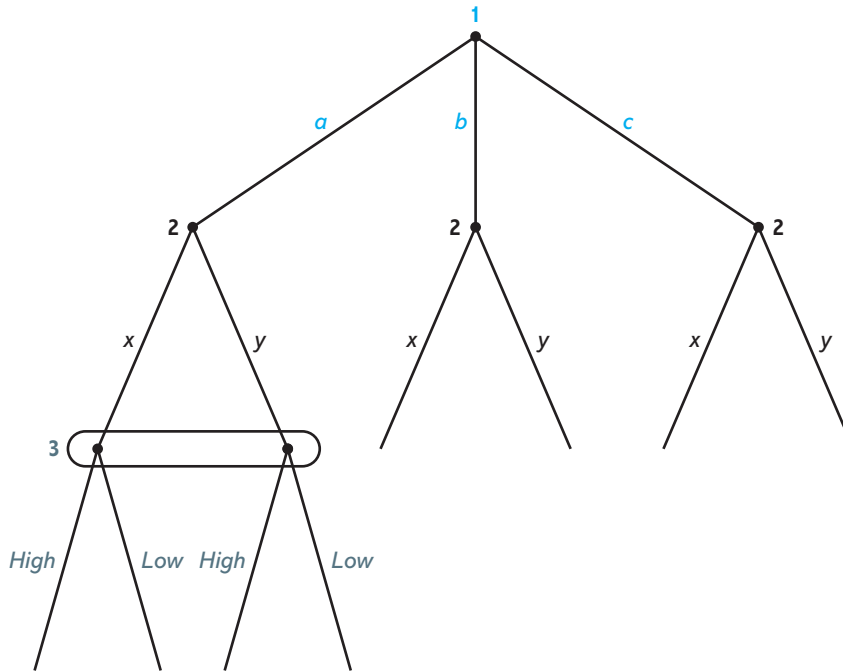
ANSWER:
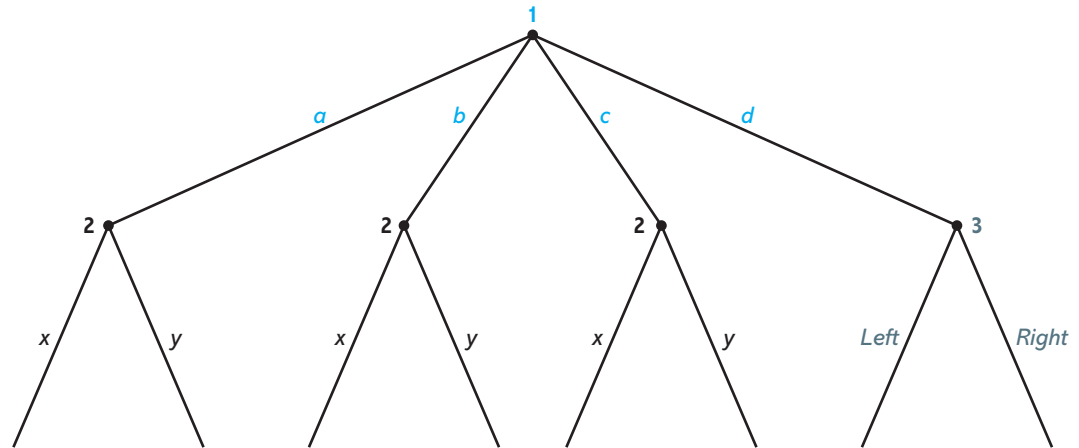
3. Consider a setting in which player 1 moves first by choosing among three actions: *a*, *b*, and *c*. After observing the choice of player 1, player 2 chooses among two actions: *x* and *y*. Consider the following three variants as to what player 3 can do and what she knows when she moves:

a. If player 1 chose *a*, then player 3 selects among two actions: *high* and *low*. Player 3 knows player 2's choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)
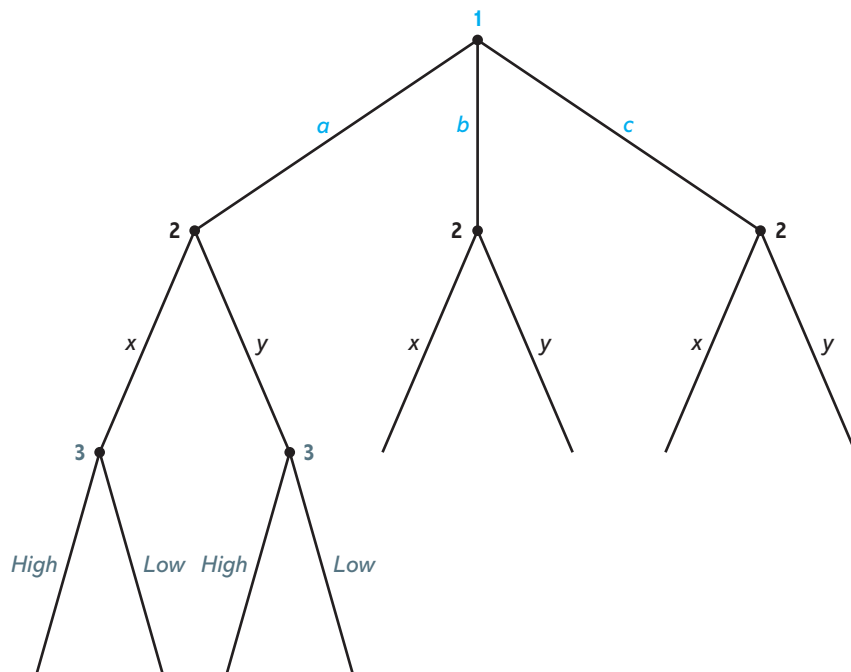
ANSWER:

b. If player 1 chose *a*, then player 3 selects among two actions: *high* and *low*. Player 3 does not know player 2's choice when she moves. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:

c. If player 1 chose either *a* or *b*, then player 3 selects among two actions: *high* and *low*. Player 3 observes the choice of player 2, but not that of player 1. Write down the extensive form of this setting. (You can ignore payoffs.)

ANSWER:
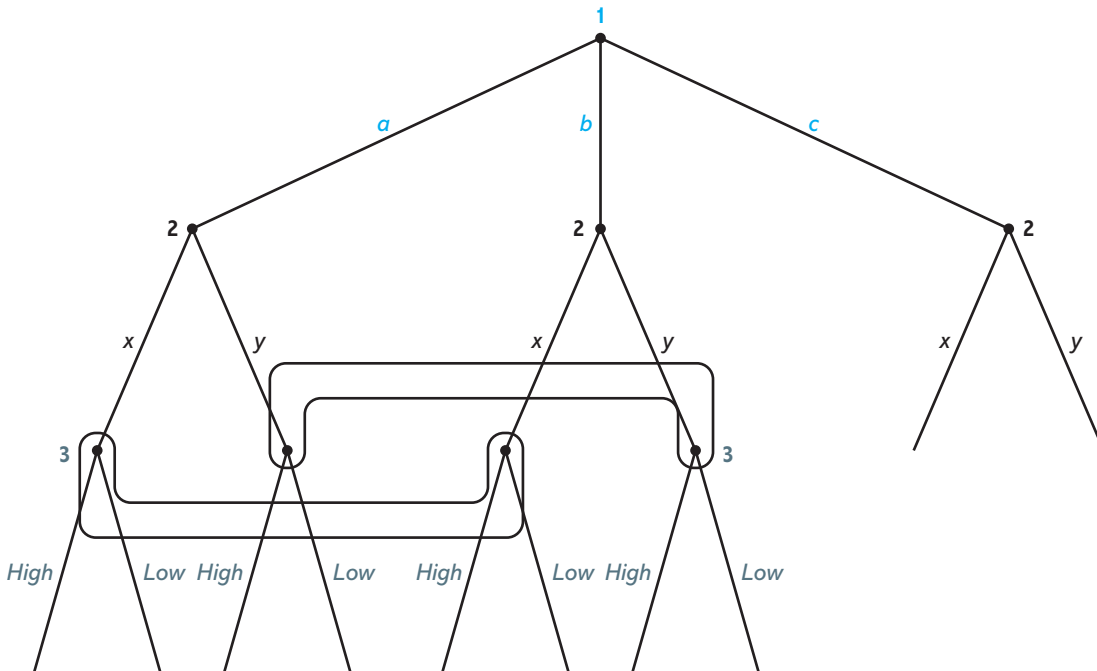
**4.** Return to the game involving the U.S. Court of Appeals in Section 2.2. Suppose, at the start of the game, it is known by all that Judge Z will read only the brief of Ms. Hasenpfeffer. Write down the corresponding extensive form game. You may exclude payoffs.

ANSWER TO 4:
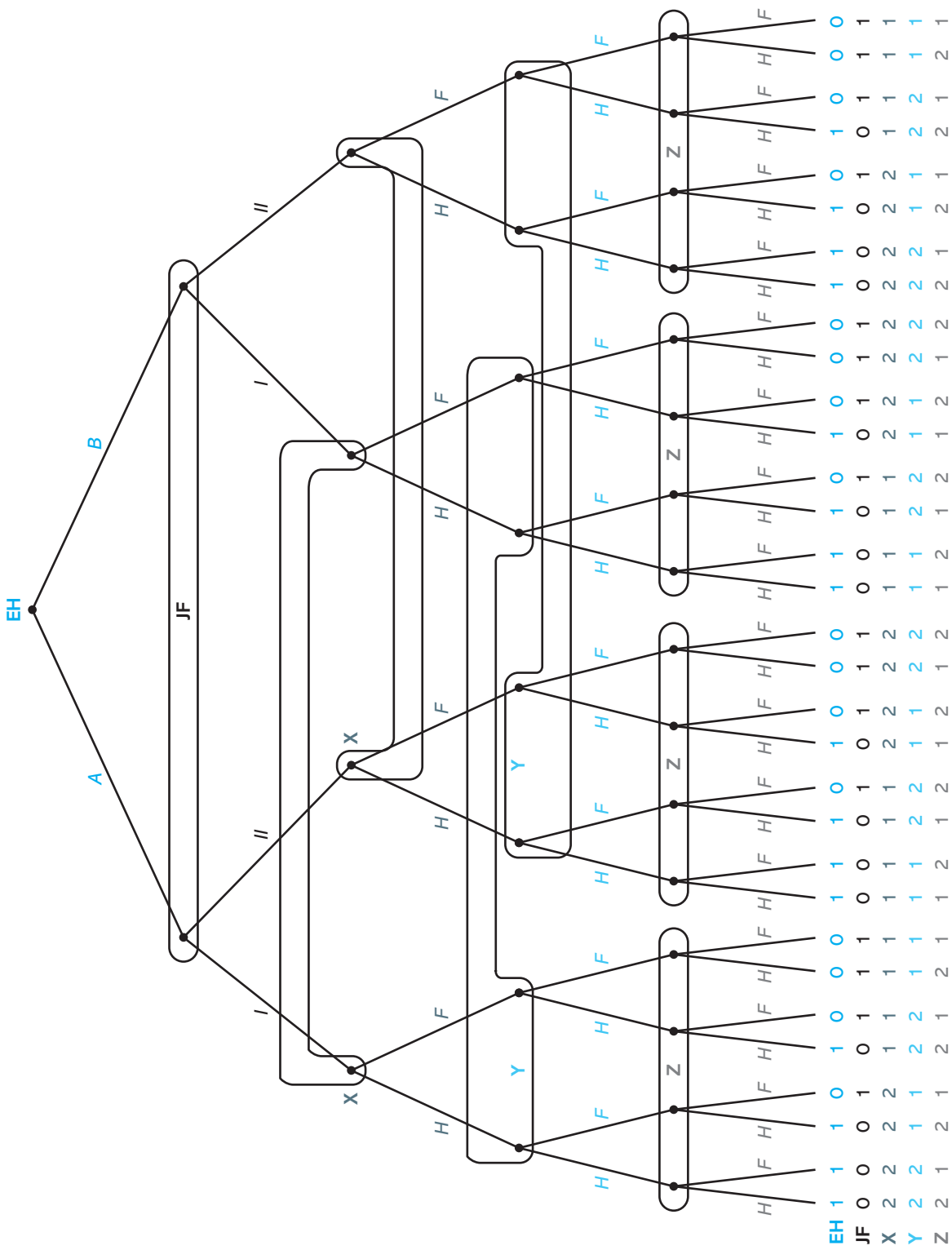
5. The city council is to decide on a proposal to raise property taxes. Suppose Ms. Tuttle is the chair and the council's other two members are Mr. Jones and Mrs. Doubtfire. The voting procedure works as follows: Excluding the chair, Mr. Jones and Mrs. Doubtfire simultaneously write down their votes on slips of paper. Each writes either *for* or *against* the tax increase. The secretary of the city council then opens the slips of paper and announces the vote tally. If the secretary reports that both slips say *for*, then the tax increase is implemented and the game is over. If both vote *against,* then the tax increase is not implemented and, again, the game is over. However, if it is reported that the vote is one *for* and one *against*, then Ms. Tuttle has to vote. If she votes *for,* then the tax increase is implemented, and if she votes *against,* then it is not. In both cases, the game is then over. As to payoffs, if the tax increase is implemented, then Mrs. Doubtfire and Mr. Jones each receive a payoff of 3. If the tax increase proposal fails, then Mrs. Doubtfire has a payoff of 4 and Mr. Jones's payoff is 1. As for Ms. Tuttle, she prefers to have a tax increase—believing that it will provide the funds to improve the city's schools—but would prefer not to be on record as voting for higher taxes. Her payoff from a tax increase when her vote is not required is 5, her payoff from a tax increase when her *for* vote is required is 2, and her payoff from taxes not being increased is zero (regardless of whether or not she voted). Write down the extensive form of the game composed of Ms. Tuttle, Mr. Jones, and Mrs. Doubtfire.

ANSWER:
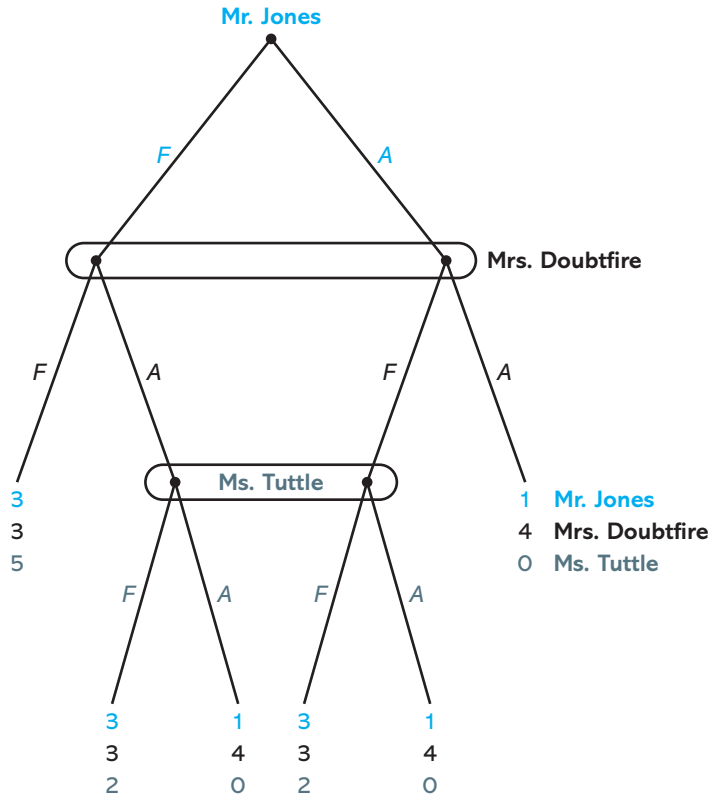
6. Consider a contestant on the legendary game show *Let's Make a Deal*. There are three doors, and behind two doors is a booby prize (i.e., a prize of little value), while behind one door is a prize of considerable value, such as an automobile. The doors are labeled 1, 2, and 3. The strategic situation starts when, prior to the show, the host, Monty Hall, selects one of the three doors behind which to place the good prize. Then, during the show, a contestant selects one of the three doors. After its selection, Monty opens up one of the two doors not selected by the contestant. In opening up a door, a rule of the show is that Monty is prohibited from opening the door with the good prize. After Monty opens a door, the contestant is then given the opportunity to continue with the door originally selected or switch to the other unopened door. After the contestant's decision, the remaining two doors are opened.

a. Write down an extensive form game of *Let's Make a Deal* up to (but not including) the stage at which the contestant decides whether to maintain his original choice or switch to the other unopened door. Thus, you are to write down the extensive form for when (1) Monty Hall chooses the door behind which the good prize is placed; (2) the contestant chooses a door; and (3) Monty Hall chooses a door to open. You may exclude payoffs.

ANSWER:
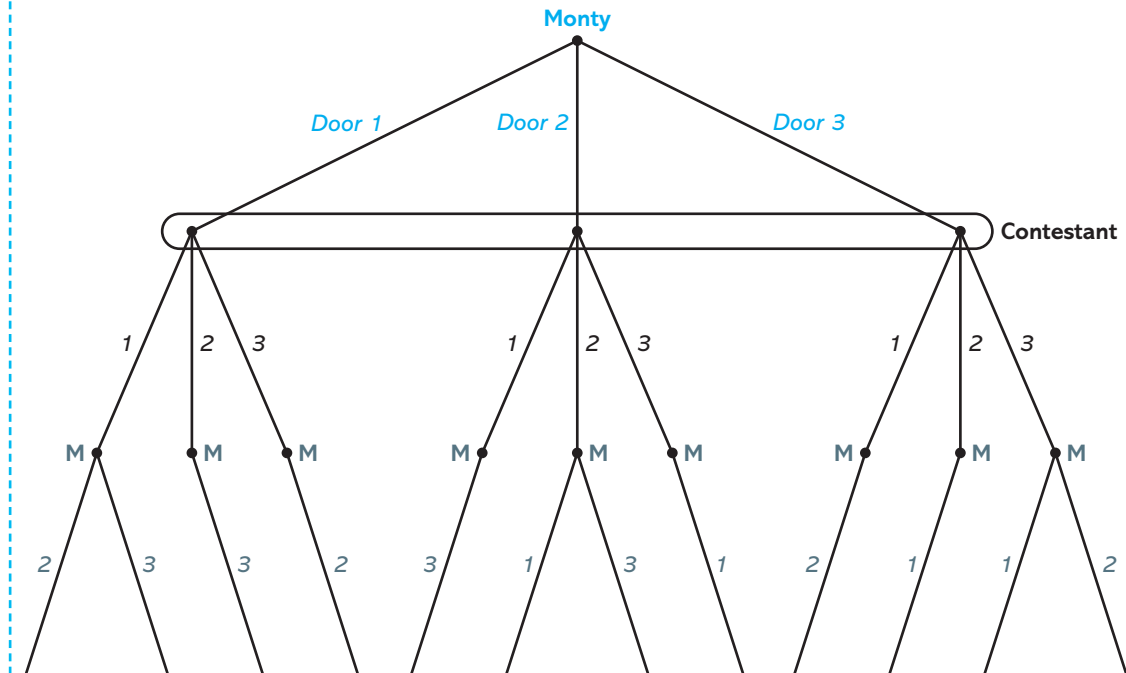
b. For the stage at which the contestant decides whether or not to switch, write down the contestant's collection of information sets. In doing so, denote a node by a triple, such as 3/2/1, which describes the sequence of play leading up to that node. 3/2/1 would mean that Monty Hall put the good prize behind door 3, the contestant initially selected door 2, and Monty Hall opened door 1.

ANSWER: There are six information sets for the contestant at the point when he has to decide whether or not to switch doors: (1) nodes 1/1/2 and 3/1/2; (2) nodes 1/1/3 and 2/1/3; (3) nodes 1/2/3 and 2/2/3; (4) nodes 1/3/2 and 3/3/2; (5) nodes 2/2/1 and 3/2/1; and (6) nodes 2/3/1 and 3/3/1. For example, the first information set comprises nodes 1/1/2 and 3/1/2. At node 1/1/2, Monty put the prize behind door 1, the contestant chose door 1, and Monty opened door 2. At node 3/1/2, Monty put the prize behind door 3, the contestant chose door 1, and Monty opened door 2. The contestant cannot discriminate between those two nodes since they entail the same sequence of observed actions—the contestant chose door 1 and Monty opened door 2—and differ only in terms of where Monty put the prize. That the information set includes both nodes 1/1/2 and 3/1/2 means that the contestant doesn't know whether the good prize is behind door 1 or door 3.

7. For the Iraq War game in Figure 2.11, write down the strategy sets for the three players.

ANSWER: Iraq has three information sets: (1) the initial node; (2) the set in which it does not have WMD and the UN requested inspections; and (3) the set in which it does have WMD and the UN requested inspections. A strategy for Iraq is then a triple of actions. At two of those information sets it has two feasible actions and at the other one it has three actions. The total number of strategies for Iraq is then 12 strategies. The United States has four information sets: (1) the UN did not request inspections; (2) the UN requested inspections and Iraq rejected the

request; (3) the UN requested inspections, Iraq acquiesced to the request, and WMD were not found (that is, either Iraq doesn't have them or has them and hid them); and (4) the UN requested inspections, Iraq acquiesced to the request,, and WMD were found (that is, Iraq had them and did not hide them). The first three information sets each comprise two nodes, one corresponding to Iraq's having WMD and one to its not having WMD. The final information set is a singleton because of the implicit assumption that a UN inspection will reveal that Iraq has WMD when Iraq does not attempt to hide them. A strategy for the U.S. is then a 4-tuple of actions. Since at each of its four information sets the U.S. has two feasible actions—attacking Iraq or not attacking Iraq—the U.S. has 16 strategies. Finally, the UN has one information set. Like the U.S., it has two nodes with one corresponding to Iraq's having WMD and one not. A strategy for the UN is then a single action; its strategy set is composed of *request inspections* and *do not request inspections*.

8. Derive the corresponding strategic form for the extensive form game in the figure below.
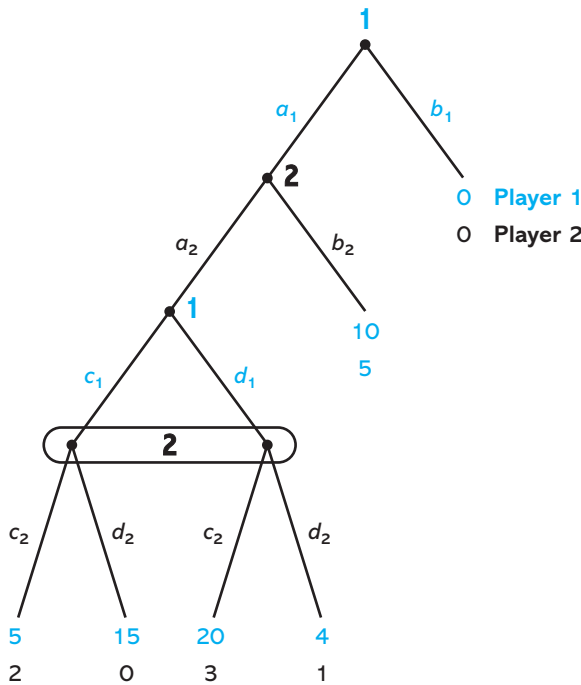
ANSWER: Player 1 has two information sets, the initial node and the information set associated with a_1 and a_2 having been played. Let x/y denote a strategy for player 1 that assigns action x to the initial node and action y to the other information set. Player 1's strategy then contains four elements: a_1/c_1, a_1/d_1, b_1/c_1, and b_1/d_1. Player 2 also has two information sets, the singleton associated with 1 having used a_1 and the information set with two nodes—one when the path is $a_1 \rightarrow a_2 \rightarrow c_1$ (read as "a_1 is chosen then a_2 is chosen then c_1 is chosen") and one when the path is $a_1 \rightarrow a_2 \rightarrow d_1$. If strategy x/y assigns action x to the first information set and action y to the second one, then player 2 has four strategies: a_2/c_2, a_2/d_2, b_2/c_2, and b_2/d_2. The payoff matrix associated with these strategies is shown in the figure below.

Player 2

		a_2/c_2	a_2/d_2	b_2/c_2	b_2/d_2
	a_1/c_1	5,2	15,0	10,5	10,5
	a_1/d_1	20,3	4,1	10,5	10,5
Player 1	b_1/c_1	0,0	0,0	0,0	0,0
	b_1/d_1	0,0	0,0	0,0	0,0

9. Write down the strategic form game for the extensive form game below.

| Player | 1 | 4 | 1 | 2 | 0 | 3 | 4 | 1 | 0 |
| Player | 2 | 2 | 3 | 2 | 6 | 1 | 2 | 5 | 0 |
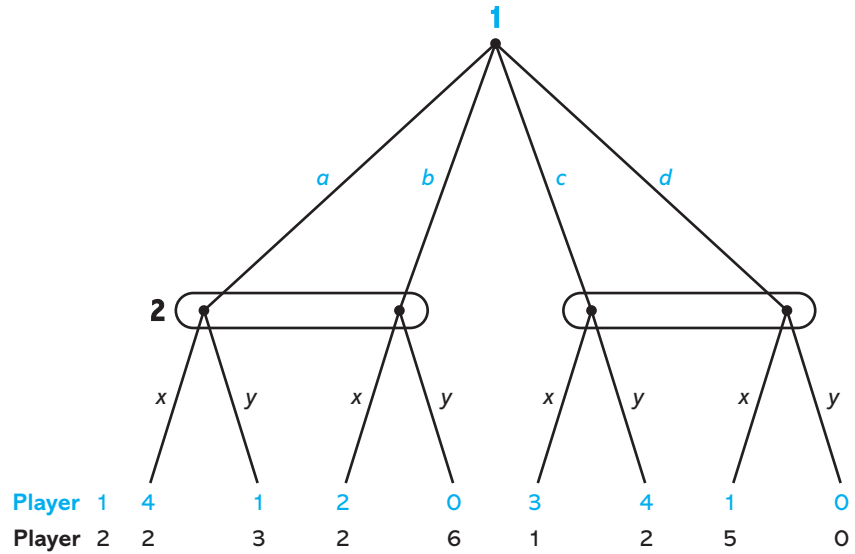
ANSWER: Player 1 has only one information set, which is the initial node. Player 2 has two information sets. Her first information set is the information set associated with player 1 having chosen either *a* or *b*. Her second information set is associated with player 1 having chosen *c* or *d*. Strategy *x/y* for player 2 assigns action *x* to the first information set and action *y* to the second information set. The strategic form game for this game is shown in the figure below.

Player 2

		x/x	x/y	y/x	y/y
	a	4,2	4,2	1,3	1,3
	b	2,2	2,2	0,6	0,6
Player 1	**c**	3,1	4,2	3,1	4,2
	d	1,5	0,0	1,5	0,0

10. Write down the strategic form game for the extensive form game in the game below.

Player	1	4	2	1	6	2	1
Player	2	1	3	1	0	3	0
Player	3	2	0	1	3	5	6
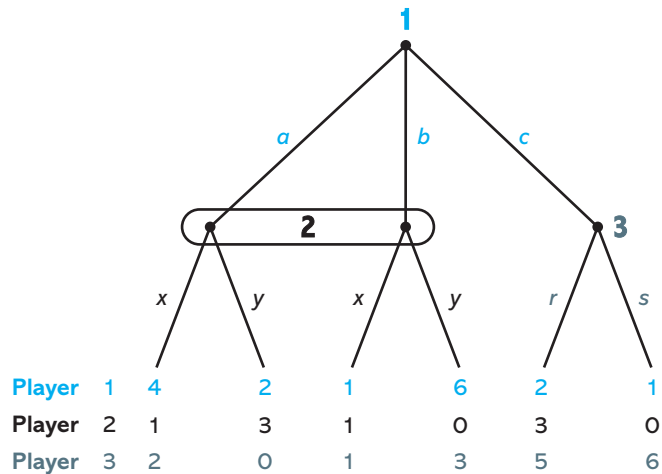
ANSWER: Each player has only one information set. The strategic form game is shown in the figure below.

Player 3: *r*

Player 2

	x	*y*
a	4,1,2	2,3,0
b	1,1,1	6,0,3
c	2,3,5	2,3,5

Player 1 labels rows a, b, c.

Player 3: *s*

Player 2

	x	*y*
a	4,1,2	2,3,0
b	1,1,1	6,0,3
c	1,0,6	1,0,6

Player 1 labels rows a, b, c.

11. Three extensive form games are shown in the following figure. State which of them, if any, violate the assumption of perfect recall. Explain your answer.

(a)

(b)
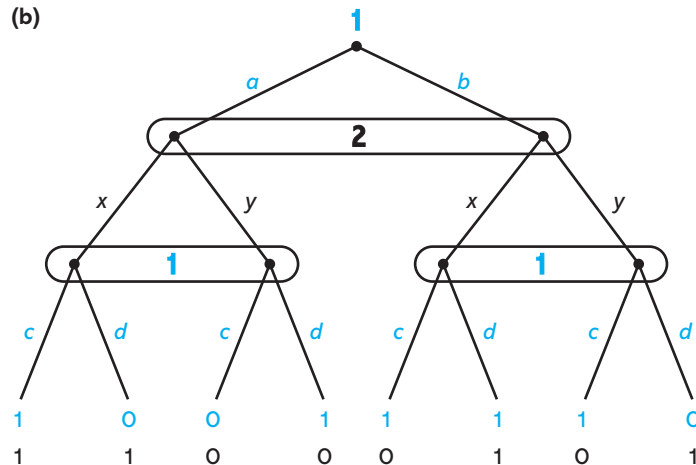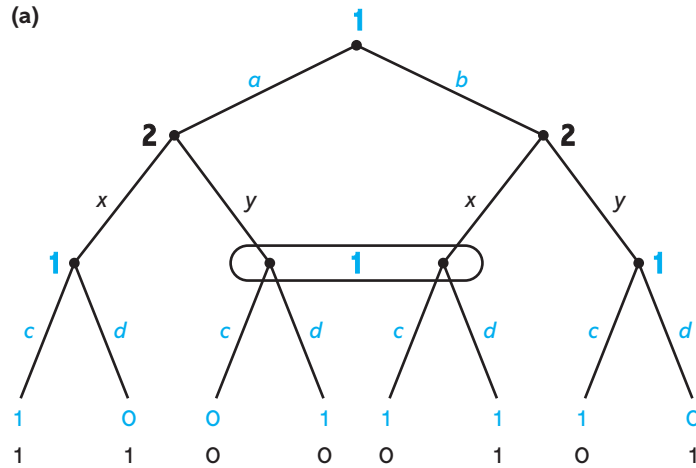
(*Continued*)   **(c)**
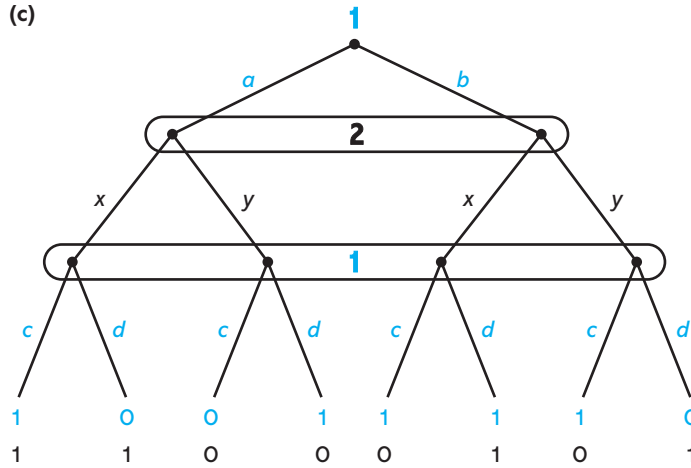
ANSWER: Only game (b) satisfies perfect recall. In game (a), consider the information set for player 1 that includes two nodes. One node is associated with 1 having chosen a and 2 having chosen y. The other is associated with 1 having chosen b and 2 having chosen x. At this information set, 1 is then unsure whether she chose a or b. That violates perfect recall. As to game (c), the information set for player 1 which includes four nodes captures the property that, when 1 chooses between actions c and d, she doesn't know what player 2 chose (which is not in violation of perfect recall) nor what she originally chose at the initial node (which is in violation of perfect recall). Game (b) satisfies perfect recall. When 1 chooses between actions c and d, she cannot discriminate between the nodes in which play was $b \rightarrow x$ and play was $a \rightarrow y$, nor between the nodes in which play was $b \rightarrow x$ and play was $b \rightarrow y$. The former reflects 1's uncertainty over 2's action but knowledge that she originally chose a. The latter reflects 1's uncertainty over 2's action but knowledge that she originally chose b.

12. Alexa and Judd live in Boston and have been dating for about a year and are fairly serious. Alexa has been promoted to Regional Manager and been given the choice of assignments in Atlanta, Boise, and Tucson. After she makes her choice (and this is observed by Judd), he'll decide whether to stay in Boston or follow Alexa. The payoffs associated with the six possible outcomes are in the accompanying table.
    a. Derive the extensive form game.

ANSWER:

Player 1: Alexa
Player 2: Judd
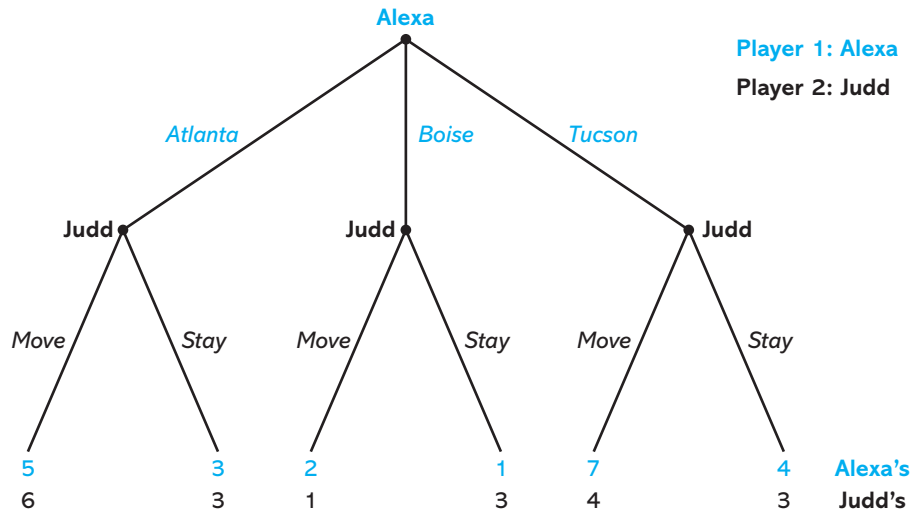
b. Derive the strategic form game.

Alexa's choice	Judd's choice	Alexa's payoff	Judd's payoff
Atlanta	Move	5	6
Atlanta	Stay	3	3
Boise	Move	2	1
Boise	Stay	1	3
Tucson	Move	7	4
Tucson	Stay	4	3

ANSWER: The strategy set of Alexa is {A,B,T}. A strategy for Judd is a 3-tuple of actions; what to do if Alexa moves to Atlanta, what to do if she moves to Boise, and what to do if she moves to Tucson. His strategy set is {M/M/M,M/M/S,M/S/M,M/S/S, S/M/M,S/M/S,S/S/M,S/S/S}.

	A	B	T
M/M/M	5,6	2,1	7,4
M/M/S	5,6	2,1	4,3
M/S/M	5,6	1,3	7,4
M/S/S	5,6	1,3	4,3
S/M/M	3,3	2,1	7,4
S/M/S	3,3	2,1	4,3
S/S/M	3,3	1,3	7,4
S/S/S	3,3	1,3	4,3

13. When he released his new novel *The Plant*, the best-selling author Stephen King chose to make early chapters downloadable for free on his website www.stephenking.com but he also asked readers to make voluntary contributions. Furthermore, he stated that he would not release subsequent chapters unless people contributed: "Remember: Pay and the story rolls. Steal and the story folds." In modeling this approach to selling a book, suppose there are just three readers: Abigail, Carrie, and Danny. All chapters have been released except for the final one, which, of course, has the climax. For Abigail or Carrie, if the final chapter is released then each receives a payoff of 5 minus how much money she contributed. For Danny, if the final chapter is released then he receives a payoff of 10 minus how much money he contributed. If the final chapter is not released then each reader receives a payoff of 2 minus how much he or she contributed. Abigail and Carrie are deciding between contributing nothing and $2. Danny is deciding between $2 and $4. For the final chapter to be released, at least $6 must be raised.

a. Assume all three readers make simultaneous contribution decisions. Write down the strategic form game.

ANSWER: The set of players is Abigail, Carrie, and Danny. The strategy set for Abigail and Carrie is {0,2}, and for Danny is {2,4}. The strategic form is shown below where the first payoff in a cell is for Abigail, the second payoff is for Carrie, and the third is for Danny.

Danny: 2

Carrie

		0	2
Abigail	0	2,2,0	2,0,0
	2	0,2,0	3,3,8

Danny: 4

Carrie

		0	2
Abigail	0	2,2,−2	5,3,6
	2	3,5,6	3,3,6

Now suppose Danny contributes first, and then Abigail and Carrie make simultaneous contribution decisions after observing Danny's contribution.

b. Write down the extensive form game.

ANSWER:

Player 1: Danny
Player 2: Abigail
Player 3: Carrie
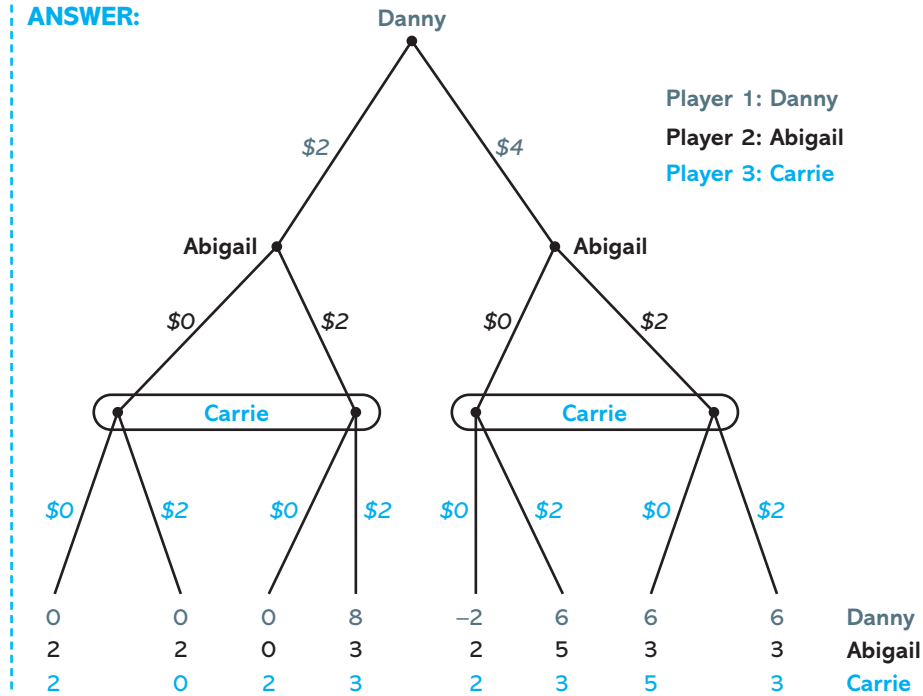
c. Write down each player's strategy set.

ANSWER: A strategy for Danny is a single action and his strategy set is {2,4}. A strategy for Abigail or Carrie is a pair of actions; what to do when Danny chooses 2 and what to do when Danny chooses 4. The strategy set for Abigail and Carrie is {0/0,0/2,2/0,2/2}.

**d.** Write down the strategic form game.

ANSWER:

Danny: 2

		Carrie			
		0/0	0/2	2/0	2/2
	0/0	2,2,0	2,2,0	0,2,0	0,2,0
	0/2	2,2,0	2,2,0	0,2,0	0,2,0
Abigail	2/0	0,2,0	0,2,0	3,3,8	3,3,8
	2/2	0,2,0	0,2,0	3,3,8	3,3,8

Danny: 4

		Carrie			
		0/0	0/2	2/0	2/2
	0/0	2,2,−2	5,3,6	2,2,−2	5,3,6
	0/2	3,5,6	3,3,6	3,5,6	3,3,6
Abigail	2/0	2,2,−2	3,5,6	2,2,−2	5,3,6
	2/2	3,5,6	3,3,6	3,5,6	3,3,6

14. There are four shoppers in a store—Amy, Brian, Cassandra, and David—who sequentially show up at two checkout lines: line number 1 and line number 2. Initially, both checkout lines are empty. Amy shows up first, sees no one in either line, and chooses one of the two lines. Brian shows up next, sees which line Amy entered, and chooses between the two lines. Next, Cassandra shows up. She sees Amy and Brian in line and chooses between the two lines. Finally, David arrives, sees Amy, Brian, and Cassandra waiting in line, and enters one of the two lines. Assume a player's payoff equals 5 minus the number of shoppers in line ahead of him or her.
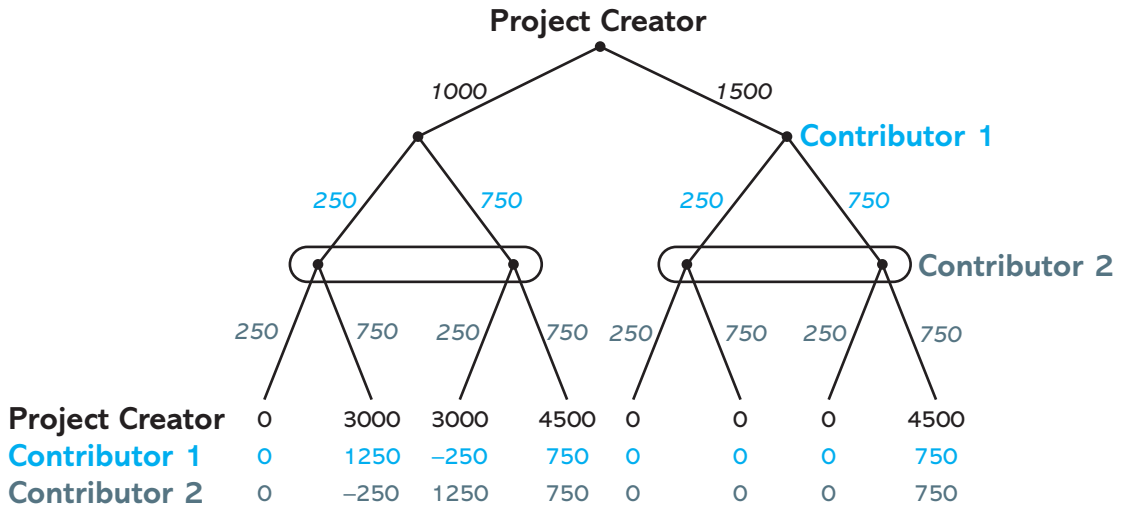
a. Derive the extensive form game.

b. Now suppose that by the time that David shows up, Amy has already checked out. Thus, only Brian and Cassandra are in line. Derive the extensive form game.

A: Amy
B: Brian
C: Cassandra
D: David

Part (a)

Part (b)
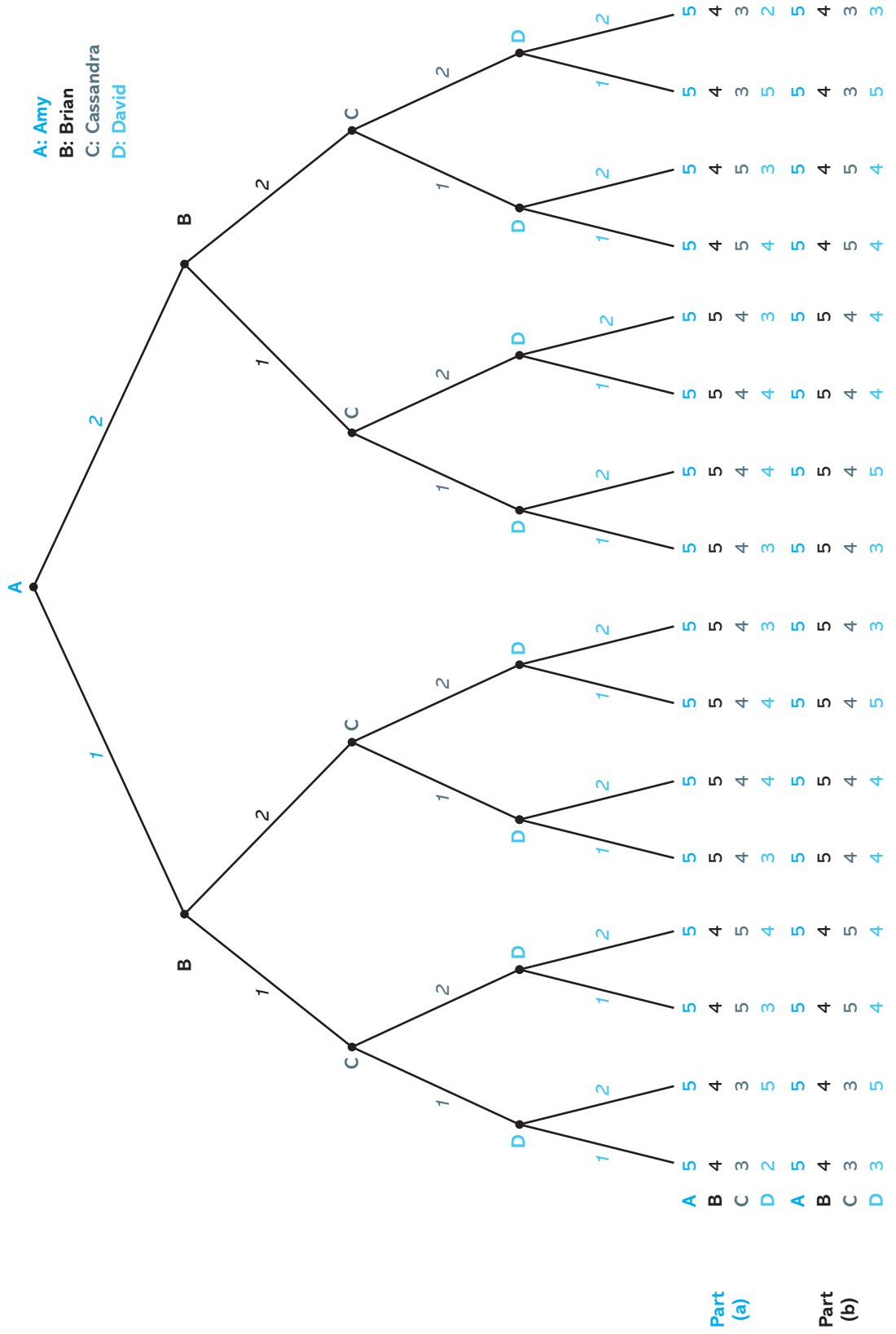
15. Kickstarter (www.kickstarter.com) provides a platform for raising venture capital through crowdsourcing. A project creator sets a funding target and posts the project at Kickstarter. People then decide how much money to pledge. If the total pledges are at least as great as the funding target, then the pledges are converted into contributions and the project is funded. Though the contributors do not own a share of the project, they can receive rewards from the project creator. If the pledges fall short, then the project is not funded. Assume there are three players: one project creator and two potential contributors. The project creator chooses between a funding target of $1,000 and $1,500. With the funding target posted at Kickstarter, the two contributors simultaneously decide whether to pledge $250 or $750. Assume the project creator's payoff equals three times the amount of funding (which is zero if contributions are less than the funding target). A contributor's payoff is zero when the project is not funded (irrespective of the pledge made), and is two times the total amount of pledges minus three times the contributor's own pledge when it is funded.

a. Write down the extensive form game.

ANSWER:

| | 250/250 | 250/750 | 750/250 | 750/750 |

Project Creator   0   3000   3000   4500   0   0   0   4500
Contributor 1   0   1250   −250   750   0   0   0   750
Contributor 2   0   −250   1250   750   0   0   0   750

b. Write down each player's strategy set.

ANSWER: The project creator's strategy is a target level of funding target and the strategy set is {1000,1500}. A contributor's strategy is a pair of actions that specifies an amount of pledge for each funding target chosen by the project creator. A contributor' strategy set is {250/250,250/750,750/250,750/750}.

c. Write down the strategic form game.

ANSWER:

Project creator: 1000

Contributor #2

		250/250	250/750	750/250	750/750
	250/250	0,0,0	0,0,0	1250,−250,3000	1250,−250,3000
	250/750	0,0,0	0,0,0	1250,−250,3000	1250,−250,3000
Contributor #1	**750/250**	−250,1250,3000	−250,1250,3000	750,750,4500	750,750,4500
	750/750	−250,1250,3000	−250,1250,3000	750,750,4500	750,750,4500

(Continued)

Project creator: 1500

Contributor #2

	250/250	250/750	750/250	750/750
250/250	0,0,0	0,0,0	0,0,0	0,0,0
250/750	0,0,0	750,750,4500	0,0,0	750,750,4500
750/250	0,0,0	0,0,0	0,0,0	0,0,0
750/750	0,0,0	750,750,4500	0,0,0	750,750,4500

(Contributor #1 labels the rows.)

16. Consider drivers who commonly traverse a major highway. Each driver is deciding whether to buy E-ZPass. E-ZPass electronically charges a driver for going through a toll, which avoids having to stop and hand over money. E-ZPass costs $4 and allows a driver to go through the E-ZPass lane. Without E-ZPass, a driver goes through the Cash lane. With either lane, the toll is $6. The average time it takes for a car to get through the E-ZPass line is 10 seconds multiplied by the number of cars in the E-ZPass lane (which is assumed to equal the number of cars with E-ZPass). For the Cash lane, the average time it takes for a car to get through is 30 seconds multiplied by the number of cars in the Cash lane (which is assumed to equal the number of cars without E-ZPass). The value of a driver's time is 30 cents per minute. Assume there are 100 drivers, each of whom has a payoff equal to 20 minus the value of time spent in line minus expenditure (the latter is $4 without E-ZPass and $10 with E-ZPass). Drivers make simultaneous decisions about whether or not to buy E-ZPass.

 a. The strategy set for a driver is (*E-ZPass, No E-ZPass*). Derive a driver's payoff function depending on his choice and the choices of the other 99 drivers.

 ANSWER: Let m denote the number of other drivers that choose E-ZPass. A driver's payoff from buying E-ZPass is $20 - .3(\frac{m+1}{6}) - 10$, and from not buying E-ZPass is $20 - .3(\frac{100-m}{2}) - 6$

 b. Now suppose a driver with E-ZPass can use either lane. Assume that it takes the same amount of time to go through the Cash lane whether a driver has E-ZPass or not. Drivers without E-ZPass can still go through the Cash-only lane. The strategy set for a driver is (*E-ZPass & E-ZPass lane, E-ZPass & Cash lane, No E-ZPass & Cash lane*). Derive a driver's payoff function, depending on her choice and the choices of the other 99 drivers.

 ANSWER: Let m denote the number of other drivers that choose E-ZPass & E-ZPass lane and n denote the number of other drivers that choose E-ZPass & Cash lane. A driver's payoff from E-ZPass & E-ZPass lane is $20 - .3(\frac{m+1}{6}) - 10$, from E-ZPass & Cash lane is $20 - .3(\frac{100-m}{6}) - 10$, and from not buying E-ZPass is $20 - .3(\frac{100-m}{2}) - 6$